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Abstract

This paper uses a recent model on ticket pricing with the aim of studying how the population of potential buyers is partitioned when there are two ranked qualities on offer. Our exercise incorporates specific distributions of tastes in the computation of the first order conditions for price selection. The role of the capacity constraints is also analysed and made endogenous.

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1. Introduction

The provision of quality by a discriminating monopolist is commonly studied under the assumption that the monopolist sells different varieties of a private good to people with homogeneous tastes but different reservation prices. In other words, customers are assumed to rank different varieties of a given good in the same way, but have different willingness to pay due to the differences in their incomes (and, thus, their ability to pay). The analysis of price and quality discrimination, typically relegated to the case of natural monopolies or public utilities, usually assumes that the production capacity and the number of customers are given.

In a recent paper on ticket pricing, Rosen and Rosenfield [3](hereafter R-R) use the theory of second degree price discrimination to analyse some commonly observed ticket pricing policies under conditions of deterministic demand. They stress that price discrimination tends to be observed in activities where capacity constraints makes the marginal cost of provision smaller than the average cost and, for this reason, is also common in highly competitive businesses like hotels, airlines, restaurants and so on, and show how the relative variance of customers' valuations between service classes affects the degree of discrimination. One of the interesting features of their analysis is the derivation of the revenue maximising gap between the prices of two qualities on sale. Under the assumption that tastes are ordered, they find the marginal conditions for the derivation of the optimal price gap and the price for the lower quality and indicate how the variables can be derived recursively. Explicit formulas for the key variables, however, cannot be derived without specifying the functional form of the distribution of tastes.

The aim of this paper is to study how an heterogeneous population of potential buyers is partitioned when there are two qualities on offer, and what is the role played by the distribution of buyers' tastes in the resulting pricing scheme.

Our first purpose is to characterise the properties of the revenue maximising gap between prices for given distribution of tastes over the two qualities. We then derive the conditions for the socially optimal gap between prices of low and high quality service. To this purpose, we use the formal analogy between the peak-load pricing problem and the supply of quality discriminated services.

The paper is organised as follows. Section 2 describes the reference model. Section 3 discusses the nature of the first order conditions, the role played by the capacity constraints and presents the equations characterising the optimal price gap and the optimal price for the lower quality for two distributions of tastes (the details of the derivation of our results are given in the first two appendixes) . Section 4 presents the results of some numerical simulations of the model.

2. Reference model

R-R analyse the problem of pricing discrete classes of services such as first and second class seats on trains or airplanes. When price discrimination is applied to quality discrimination, prices are used to sort and select customers on the basis of their tastes. This section sketches the basic model by R-R. The solution for the one-factor representation of tastes is derived in detail, since it is the basis of our analysis.

A revenue maximising firm offers a good or a service whose characteristics allow for price discrimination. The firm has to set the prices of the different service classes in order to get the desired number of customers for each class, given a capacity constraint.

The good can be sold at two given qualities, high (H) and low (L). All customers are assumed to rank different qualities in the same order, but their willingness to pay for each quality is not the same since it depends on the intensity of demand that individuals have for each quality. Individual preferences are completely described by the reservation prices attached to the two qualities: r_H and r_L. The choice between quality H or L is taken according to

$$\max(r_H - p_H, r_L - p_L, 0), \tag{2.1}$$

where p_H and p_L are the prices of qualities H and L. Reserve prices for each quality depend on the intensity of demand (T) that each customer has for a given quality, as in

$$r_j = \alpha_j + \beta_j T \tag{2.2}$$

with $j = L, H$, and α_j, β_j are fixed parameters and β_j is strictly positive. Solving for T, prices are distributed over a positively sloped line in the (r_H, r_L) plane:

$(\beta_L r_L + \alpha_L \beta_H - \beta_H r_L - \alpha_H \beta_L = 0)$.

The intensity of demand T has a cumulative distribution, $G(T) = \int_0^T g(t)\,dt$ with $G(\infty) = N$, where N is the number of people with positive taste for either type of service. The demand functions are derived by calculating the cut off points (T_0, T_1) separating customers who choose H from those choosing L. T_0 and T_1 obey

$$T_0 : \begin{cases} r_L = p_L \\ r_L = \alpha_L + \beta_L T \end{cases} \tag{2.3}$$

$$T_1 : \begin{cases} r_H - p_H = r_L - p_L \\ \beta_L r_L + \alpha_L \beta_H - \beta_H r_L - \alpha_H \beta_L = 0 \end{cases} \tag{2.4}$$

so that

$$\begin{cases} T_0 = \dfrac{p_l - \alpha_l}{\beta_l} \\ T_1 = \dfrac{(p_H - p_l) - (\alpha_H - \alpha_l)}{(\beta_H - \beta_l)} \end{cases} \tag{2.5a}$$

Demands for the two qualities depend on p_H, p_L and Δp $(p_h - p_l)$ since

$$N_H = N - G(T_1) \text{ and } N_L = G(T_1) - G(T_0). \tag{2.6}$$

The objective of the firm is to maximise total revenues:

$$\begin{aligned} & \max(p_H N_H + p_L N_L) \\ = \ & \max(p_H (N - G(T_1)) + p_l (G(T_1) - G(T_0))) \\ = \ & \max((p_H - p_L)(N - G(T_1)) + p_L (N - G(T_0))) \\ = \ & \max(\Delta p (N - G(T_1)) + p_L (N - G(T_0))) \end{aligned}$$

where $G(T_1)$ depends on Δp and $G(T_0)$ on p_L. Given $N_H \leq \bar{N}_H$ and $N_L \leq \bar{N}_L$ as capacity constraints, the marginal conditions for Δp and p_L are

$$\Delta p : \begin{cases} \dfrac{\partial}{\partial \Delta p}\left[p_H N_H + p_L N_L\right] = \\ = \dfrac{\partial}{\partial \Delta p}\left[\Delta p\left(N - G\left(T_1\right)\right) + p_L\left(N - G\left(T_0\right)\right)\right] \end{cases}$$

which is, since $\frac{\partial}{\partial \Delta p}\left[\Delta p\left(N - G\left(T_1\right)\right)\right] \equiv \frac{\partial}{\partial \Delta p}\left[\Delta p N_H\right]$,

$$N_H + \Delta p \frac{\partial N_H}{\partial \Delta p} > (=) 0. \tag{2.7}$$

which holds as an inequality(equality) if the capacity constraint is binding (not binding) (i.e. if $N_H = (<)\ \overline{N}_H$). With a similar reasoning, the first order conditions for p_L :

$$\begin{aligned} \frac{\partial}{\partial p_L}\left[p_L\left(N - G\left(T_0\right)\right)\right] &\equiv \frac{\partial}{\partial p_L}\left[p_L\left(N_H + N_L\right)\right] = \\ \left(N_H + N_L\right) + p_L \frac{\partial\left(N_H + N_L\right)}{\partial p_L} &= \left(N_H + N_L\right) + p_L \frac{\partial\left(N - N_0\right)}{\partial p_L} \end{aligned}$$

give us

$$\left(N_H + N_L\right) + p_L \frac{\partial\left(N_H + N_L\right)}{\partial p_L} = \left(N_H + N_L\right) + p_L \frac{\partial\left(N - N_0\right)}{\partial p_L}$$

$$\begin{aligned} \left(N_H + N_L\right) - p_L \frac{\partial N_0}{\partial p_L} &> (=) 0 \\ N_L &= (<) \overline{N}_L, \end{aligned} \tag{2.8}$$

i.e., also equation(2.8) holds with equality if the capacity constraint is not binding.

3. Open problems

Equation (2.7) alone determines the gap between prices, Δp, and the whole problem can be solved recursively. Exogenous parameters are those of tastes and the capacity available. Equation (2.7), however, depends on exogenous parameters only implicitly, and no general statement on the optimal solution can be given without specifying the distribution function of the taste parameter. In addition, both equation (2.7) and (2.8) are transcendental functions and, therefore, not analytically solvable: they can only be solved numerically. Since Δp is the key variable to be found by the firm, we first address the problem of how to solve equation (2.7) with respect to Δp.

The capacity constraints play an important and under explored role.If the available capacity of high quality service is slack (i.e. eq.(2.7) holds with equality), the solution for Δp from (2.7) is the first input of the algorithm . In this first case, the seller can find the maximum number of H type buyers and evaluate the excess of the capacity he has for service H.

If, alternatively, the capacity constraint is binding (eq.(2.7) holds with inequality), optimal Δp is the result of the recursion. This second case enables the comparison of the capacity constraint with the maximum number of people ready to buy H and determines the number of H type buyers rationed.[1]

4. Numerical solutions

The recursion requires Δp as the first input. Given $\Delta\alpha$, $\Delta\beta$ and the parameters of the distribution of tastes, Δp solves T_1 . From T_1 , $G(T_1)$ $(= N_0 + N_L)$ can be calculated, and (consequently) N_H . Define γ as the proportion of customers which can be served with the available capacity. If $N_H > \gamma N$, $G(T_1)$ has to equal γN, so that T_1 and Δp have to be adjusted accordingly. The same applies to p_L.

In solving the problem numerically, we have assumed the following.

[1] The same applies to the price of the low quality service.

Assumption 1.

$\alpha_H > \alpha_L$ ($\Delta\alpha > 0$): a higher r_H is also due to a higher α_H.

Assumption 2.

$\beta_H > \beta_L$. Customers with a higher reservation price for each quality have a greater "factor loading" parameter. This assumption forces the direction of the recursion.

Assumption 3.

$p_L = 1$. The price of the low quality service is normalised to one. This assumption facilitates the interpretation of Δp as the percentage increase from the low-quality to the high-quality price. This assumption poses some restrictions on α_L and β_L ($0 < \alpha_L < 1$ and $\beta_L > 0$ because of eq. (2.5a)) and forces the recursion towards N_L, while making interpretation easier. This assumption is not restrictive in any sense, and it will be omitted later in the analysis.

Assumption 4.

$g(t) \sim N(\mu, \sigma^2)$. The variable t ('taste' for the good, or intensity of demand) is normally distributed. The reason for starting with this distribution is its generality, since the normal distribution is the limit distribution of many others . Because Δp is in the upper bound of the integral in (4.1), the normally distributed $g(.)$ allows for the use of a special function, the 'error function', particularly useful for our purpose, as it can be seen in the derivation of the numerically solvable equation in Appendix 1.

The use of the normal distribution, defined also for negative values of the taste parameter, required a further assumption to avoid the possibility of assigning negative taste to a part of the potential buyers. In order to make the negative values negligible, we assumed a variance of the taste parameter smaller than the average of the parameter.

4.1. Analysis

The first step of the recursion is to solve

$$\Delta p = -\left(\frac{N_H}{\partial N_H / \partial \Delta p}\right) = \left(\frac{N - G(T_1)}{\partial \left(N - G(T_1)\right) / \partial \Delta p}\right)$$
$$= -\frac{N - \int_0^{T_1} g(t)dt}{\frac{\partial}{\partial \Delta p}\left(N - \int_0^{T_1} g(t)dt\right)}. \tag{4.1}$$

Take equation (4.1). In order to find Δp we have to solve

$$\Delta p + \frac{N - \int_0^{T_1} g(t)dt}{\frac{\partial}{\partial \Delta p}\left(N - \int_0^{T_1} g(t)dt\right)} = 0 \tag{4.2}$$

which is

$$\Delta p = -\frac{N - \left(\int_0^{\frac{\Delta p - \Delta \alpha}{\Delta \beta}} g(t)dt\right)}{\frac{\partial}{\partial \Delta p}\left(N - \int_0^{\frac{\Delta p - \Delta \alpha}{\Delta \beta}} g(t)dt\right)} \tag{4.3}$$

where

$$g\left(t\right) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\left(-\frac{1}{2}\left(t-\mu\right)^2\right)\frac{1}{\sigma^2}\right).$$

if assumption 4 holds, or

$$g\left(t\right) = \frac{1}{\Gamma(\tau)\xi^{x}} x^{\tau-1} e$$

if assumption 4bis holds. Under assumptions 1, 2 and 4 we can show that

Proposition 4.1. *The transcendental implicit function to be numerically solved for Δp is*

$$\Delta p - \frac{N - \left(\int_{-\frac{\mu}{\sigma}}^{\left(\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta}\right)-\mu\right)\frac{1}{\sigma}} \exp\left(-\frac{1}{2}z^2\right) dz\right) \frac{1}{\sigma\sqrt{2\pi}} N}{\frac{1}{\sigma^2 \Delta\beta\sqrt{2\pi}} \exp\left(\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta}\right) - \mu\right)^2 \frac{1}{\sigma^2} N} = 0 \tag{4.4}$$

where the function to be integrated is a standardised normal distribution. The derivation of equation (4.4) is shown in Appendix A. Note that the factor N multiplying the denominator and the second term of the numerator is a normalising factor: the area described by the integral is related to the probability that a certain group of people will be served with the high quality service. It is therefore a number between zero and one. What we want the density function to describe instead, is the entire population of customers to be served (or excluded) by the firm. Thus, by multiplying the density by the number of people, we actually find the proportion of customers whose reservation price makes them choose the high quality service.

As before, equation (2.8) is numerically solvable from

$$p_L = \frac{(N_H + N_L)}{\frac{\partial N_0}{\partial p_L}} = \frac{N - \int_0^{T_0} g(t)\, dt}{\frac{\partial}{\partial p_L}\left(N - \int_0^{T_0} g(t)\, dt\right)}$$

As long as Assumption 3 holds, we do not need to compute it and we can focus our attention on the optimal price gap.

Under assumptions 1, 2 and 4bis we can show that

Proposition 4.2. *When $T \sim \Gamma(\tau, \xi)$, the transcendental implicit function to be numerically solved for Δp is*

$$\Delta p - \frac{\Gamma(\tau)\,\xi^{\tau} - \int_0^{\frac{\Delta p - \Delta\alpha}{\Delta\beta}} \frac{x^{\tau-1} e^{-\frac{x}{\xi}}}{\Gamma(\tau)\,\xi^{\tau}} dx}{\frac{(\Delta p - \Delta\alpha)^{\tau-1}}{\Delta\beta^{\tau}} e^{-\frac{\Delta p - \Delta\alpha}{\xi\Delta\beta}}} = 0. \tag{4.5}$$

Equation (??) is derived in Appendix B.

4.2. Results

Appendix C shows the results of the numerical simulation.[2] According to the results, we can state the following.

[2] The algorithm has been programmed and solved with Fortran 77.

- $T \sim N(\mu, \sigma)$.

Observation 1: Δp increases with $\Delta\alpha$ and $\Delta\beta$.

The optimal gap between prices is the greater the higher the distance between fixed parameters which transform the taste for the good into reservation prices for the two qualities. This result affects the response of T_1 to $\Delta\alpha$ and $\Delta\beta$ which is not as simple as in (2.5a), because Δp varies with $\Delta\alpha$ and $\Delta\beta$. The optimal T_1 still decreases with $\Delta\alpha$, but much slower than in (2.5a). Unlike in (2.5a), T_1 increases with $\Delta\beta$. Consequently, the number of high quality service units increases with $\Delta\alpha$ and $\Delta\beta$.

Observation 2: T_1 is always lower than μ

Another general result is the relation between T_1 and the average value of the taste parameter μ. When tastes are symmetrically distributed around the mean, the revenue maximizing $T_1$is such that more than half of the population of people with positive taste for the good are induced to buy the H good. It should be remembered that T_1 is the lowest cut off point dividing H from L buyers.

The observed relation between T_1 and μ has an important implication. In the setting we have outlined, a seller facing a normally distributed g(t) can avoid the derivation of the first order condition when $\gamma N \leq \mu$. The simple solution

$$T_1 : \int_0^{T_1} g(t)dt = \gamma N$$

is optimal, and the gap between prices can be derived accordingly.

Observation 3: The greater $\Delta\beta$, the more N_H increases with $\Delta\alpha$.

Observation 4: The greater μ the less N_H increases with $\Delta\alpha$.

This result helps us seeing that results are not only related to the variance of consumers' tastes (as R-R note), but also to the mean of them. If tastes are normally distributed, a higher mean lowers Δp and N_H diminishes consequently. Therefore, if the taste parameter is higher on average, revenues are maximised by selling a smaller amount of H service units.

Observation 5: $\forall$ $\Delta\alpha$, Δp and T_1 are the greater the greater $\Delta\beta$ and μ.

Observation 6: $\forall\ \Delta\beta$, Δp is the greater the greater $\Delta\alpha$ and μ. $\forall\ \Delta\beta$, T_1 is higher (smaller) as μ ($\Delta\alpha$) increases.

- $T \sim \Gamma(\tau, \xi)$.

Observation 7: Δp and T_1 increase with ξ. N_H decreases with ξ.

Keeping τ constant, the optimal price gap increases with ξ. A higher ξ implies a lower mode of the distribution, a lower mean (because $E(T) = \frac{\tau}{\xi}$) and a greater area under the right part of the distribution. Thus, the higher the number of people with a high intensity of demand the larger the price gap. Expectedly, the firm can exploit the higher number of H-type potential buyers by enlarging Δp.

Observation 8: $\forall\tau$, there is a $\xi : T_1 > \frac{\tau}{\xi}$,
where $\frac{\tau}{\xi}$ is the mean of the Γ distribution function.

Observation 9: Δp increases with τ.

The last two observations should be compared with those involving μ in the case of normally distributed tastes: a change in one of the two parameters of the Γ distribution function implies a change in the mean of the distribution. On the other hand, the interpretation of the two distribution functions' response to their parameters is not the same to our purpose. A higher μ means that, on average, people have a higher intensity of demand. A higher τ (keeping ξ constant) lowers the skewness of the distribution and, thus, not only implies that the average individual has a higher intensity of demand (as in the previous case) but also that the number of people with intensity of demand lower than the mean decreses. In the case of non-symmetrically distributed tastes, T_1 is lower or higher than the mean of the distribution of tastes according to the other parameters' values.

Observation 10: Δp increases with $\Delta\alpha$ and $\Delta\beta$.

This is about the behavior of the optimal Δp with respect to the differences between the parameters transforming the intensity of demand into reservation prices for the two qualities. Note that the skewed distribution of tastes does not alter the response of Δp to $\Delta\alpha$ and $\Delta\beta$ observed for a symmetric distribution of tastes.

Observation 11: T_1 decreases with $\Delta\alpha$, and increases with $\Delta\beta$.

Like in the case of normally distributed tastes, the optimal T_1 decreases with $\Delta\alpha$ (but not linearly as in (2.5a)). Still, unlike in (2.5a)), T_1 increases with $\Delta\beta$ because the increase in Δp caused by an increase in $\Delta\beta$ more than compensate the lowering on T_1 from a higher $\Delta\beta$ we would expect by (2.5a).

5. Optimal capacity

In the R-R model, the first order conditions in (2.7) and (2.8) hold with inequality (equality) if the capacity constraints are binding (not binding). If capacity is exogenously given, the revenue maximising firm compares the capacity available with the maximum number of customers ready to buy the high quality service. At the beginning of the last section we defined the capacity as γN, where γ $(0 < \gamma < 1)$ is the proportion of potential buyers (N) which can be served by the available H type service units and $(1 - \gamma)$ is the residual capacity of L type service units. The firm's problem is now to compare γN with N_H. If the optimal minimum Δp implies an N_H greater than γN, Δp has to be increased to ration H type customers.

Later in their analysis, R-R relax the exogeneity of capacity and derive conditions for profit maximising capacity. In this case, capacity costs are obviously not fixed and a profit function (instead of a revenue function) has to be maximised, since the seller's problem is to reconfigure the available capacity and in doing so positive marginal capacity costs enter the analysis.

The aim of this section is to find the socially optimal capacity, namely the capacity which maximises consumers' surplus. Our problem is now to find the socially optimal gap between prices of low and high quality service such that capacity is optimal.

Before answering this question, consider the analogy between this problem and that of peak load pricing of non-storable commodities whose demand fluctuates over time.

Firstly, we are considering pricing problems related to services (like theatre performances, transport facilities and also, as we will observe later, educational activities, schools, and health care services). Services are, by definition, non storable. Secondly, off-peak and on-peak demands can be interpreted as demands for ranked varieties of a service, namely demands for services offered at different qualities. Thirdly, as long as tastes are known, demand fluctuations over time are certain and not random. The last and more delicate point is why we choose to associate the analysis of demand for different qualities to that of intertemporal instead of spatial demand fluctuations. In a spatial framework, capacity has to be equal to aggregate demand. When demands differ over time, however, it is sufficient that the capacity is adjusted to peak demand, as the higher capacity will be adequate to satisfy off-peak demand. Consider the following example. Think of a school where students have the choice of applying for the base number of schooling hours only, or for more hours including additional activities (sports, language courses, etc.) or services (e.g. a school bus). Demand for schooling hours is the same for everybody, whereas the price for the "advanced package" determines the gap between low and high demanders.

Let c and $(c+\Gamma)$ be the marginal costs for L and H, and let them be constant, for simplicity. The unconstrained optimum would require prices for the two qualities to be equal to the corresponding marginal costs, and the gap between prices should be equal to Γ. Optimal corresponding capacity would be determined by $p_H = (c + \Gamma)$, and total costs would be covered by definition.

This solution can continue to hold when the demand for low quality is small relative to the demand for high quality but, when the opposite is true, total capacity costs cannot be recovered. (Phlips, 1983, Steiner, 1957)

"Demand for the two qualities combined can be satisfied at a cost $2c$ per combined unit up to the capacity limit, and $(2c + \Gamma)$ per combined unit beyond that limit" (Phlips, pg 139).[3] Demands have then to be

[3] Phlips's analysis does not refer to demands for different qualities but to shifting demands over time.

vertically added: the vertical sum compared with $(2c + \Gamma)$ gives the point at which customers are ready to pay for the combined unit of both low and high quality.

Demands are those in equation (2.6). Vertical sum of the two, resulting from $p_L + \Delta p$, is to be compared with $(2c + \Gamma)$ in order to find the $G(T_0)$ which determines the optimal capacity. T_0 can only be calculated from a given distribution of tastes $g(t)$. Optimal Δp will be such that $N_H = N_L$, i.e. such that

$$\int_{T_0}^{T_1} g(t)dt = \int_{T_1}^{\infty} g(t)dt. \tag{5.1}$$

6. Further comments and concluding remarks

In this paper we have studied the sensitivity of the revenue maximising gap between two varieties of a private good to the distribution of consumers' tastes for the two qualities. Unlike the settings where customers are ex ante divided into homogeneous groups (and where each group has the same reservation price for the two qualities),.we have considered a case in which all customers agree on the ranking of qualities, but their reservation prices for the two qualities are different. We have focused on the fact that the model by Rosen and Rosenfield has not a general analytical solution for the gap between the two qualities' prices. Therefore, we have derived two numerically solvable expressions for it. The numerical solutions helped us to identify the regularities of the model's solutions and the responsiveness of the variables to the parameters beyond the analytical appearance, as in the case of the sensitivity of T_1 to changes in $\Delta\beta$.

One of the reasons why we have found this exercise interesting is the potential of the analysed model to be re-interpreted as referred to a distribution of income. The replacement of the distribution of tastes with an income distribution implies a different interpretation of $\Delta\alpha$ and $\Delta\beta$. Reservation prices become a linear transformation of personal income, and $\Delta\beta$ can be interpreted as the proportion of income that people are ready to devote to the different qualities of a good or a service. To give an example, we can imagine that each person is ready to spend a higher part of his or her income on a higher quality medical assistance. This modification in the interpretation is sufficient to reinterpret our results as referred to individuals with different incomes.

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[3] Rosen S., Rosenfield R., 1997, "Ticket pricing", Journal of Law and Economics, XL, 351-376.

[4] Wilson R., 1993, Non linear pricing, Oxford University Press.

7. Appendix A: tastes are normally distributed

$$\begin{cases} \Delta p - \dfrac{N - \left(\int_0^{\frac{\Delta p - \Delta \alpha}{\Delta \beta}} g(t) dt \right)}{\frac{\partial}{\partial \Delta p} \left(N - \int_0^{\frac{\Delta p - \Delta \alpha}{\Delta \beta}} g(t) dt \right)} = 0 \\ g(t) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{1}{2} (t - \mu)^2 \frac{1}{\sigma^2} \right) \end{cases} \tag{7.1}$$

Define

- $x = \Delta p$
- $y(x) = \frac{\Delta p - \Delta \alpha}{\Delta \beta} = \frac{1}{\Delta \beta} x - \frac{\Delta \alpha}{\Delta \beta} = T_1$
- $z = \frac{t - \mu}{\sigma}$
- $g(z) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{z^2}{2} \right)$,

so that we can reformulate (7.1)as

$$x = -\frac{N - \int\limits_0^{\overline{y}(x)} g(z) dz}{-\frac{\partial}{\partial x} \int\limits_0^{\overline{y}(x)} g(z) dz} \tag{7.2}$$

where the lower and upper bounds of the integral has been changed from

$$\begin{cases} z(t = 0) = -\frac{\mu}{\sigma} \\ z\left(t = \frac{\Delta p - \Delta \alpha}{\Delta \beta} \right) = \left[\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta} \right) - \mu \right] \frac{1}{\sigma} \equiv \overline{y}(x). \end{cases}$$

Equation (7.2) is then

$$x - \frac{N - \int\limits_{-\frac{\mu}{\sigma}}^{\left[\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta}\right) - \mu\right]\frac{1}{\sigma}} g(z) dz}{\frac{\partial}{\partial x} \int\limits_{-\frac{\mu}{\sigma}}^{\left[\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta}\right) - \mu\right]\frac{1}{\sigma}} g(z) dz} = 0. \tag{7.3}$$

Simplify the denominator of the second term of the last equation as

$$-\frac{\partial}{\partial x}\int_{0}^{\frac{\mu}{\sigma}} g(z)\,dz + \frac{\partial}{\partial x}\int_{0}^{\overline{\nu}(x)} g(z)\,dz = \frac{\partial}{\partial x}\int_{0}^{\overline{\nu}(x)} g(z)\,dz$$

since the first integral does not depend on x.

$$x - \frac{N - \int_{-\frac{\mu}{\sigma}}^{\left[\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta}\right)-\mu\right]\frac{1}{\sigma}} \exp\left(-\frac{z^2}{2}\right) dz \frac{1}{\sigma^2\sqrt{2\pi}}}{\frac{\partial}{\partial x}\left(\int_{-\frac{\mu}{\sigma}}^{\left[\left(\frac{\Delta p - \Delta \alpha}{\Delta \beta}\right)-\mu\right]\frac{1}{\sigma}} \exp\left(-\frac{z^2}{2}\right) dz\right)} = 0 \qquad (7.4)$$

Define an error function as

$$\text{erf}[w] = \frac{2}{\sqrt{\pi}}\int_0^w \exp(s^2)ds. \qquad (7.5)$$

For this function the derivatives are

$$\frac{d^{n+1}}{dw^{n+1}}\text{erf}[w] = (-1)^n \frac{2}{\sqrt{\pi}} H_n(w)e^{-w^2} \qquad (7.6)$$

with $n = 1, 2, ..$ and H_n is the Hermite polynomial. The first derivative of the error function is

$$\frac{d}{dw}\text{erf}[w] \equiv \frac{d^{0+1}}{dw^{0+1}}\text{erf}[w] = (-1)^0 \frac{2}{\sqrt{\pi}} H_0(w)e^{-w^2} = \frac{2}{\sqrt{\pi}}e^{-w^2}$$

since $H_0(w) = 1$.

By expressing the denominator of (7.4) as an error function, we can easily get its derivative. In doing so, we have to remember that we changed variables from $\int_0^{\nu(x)} g(t)dt$ to $\int_0^{\overline{\nu}(x)} g(z)dz$. Let $s = \frac{z}{\sqrt{2}}$ (so that $ds = dz/\sqrt{2}$) ,

$$\int_0^{\overline{\nu}(x)} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(z^2/2\right) dz$$

becomes

$$\frac{1}{\sigma\sqrt{\pi}}\int_0^{\frac{\overline{y}(x)}{2}} \exp\left(-s^2\right) ds$$

which is, when $z = \overline{y}(x)$ and $s = \overline{y}(x)/\sqrt{2}$, equal to $\frac{1}{2\sigma}\mathrm{erf}[\overline{y}(x)/\sqrt{2}]$.
By comparing

$$\int_0^{\overline{y}(x)} \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)dz \text{ with } \frac{1}{2\sigma}\,\mathrm{erf}\left[\frac{\overline{y}}{\sqrt{2}}\right] \tag{7.7}$$

we see that what we have to solve is

$$-\underbrace{\frac{\partial}{\partial x}}\left(\frac{1}{2\sigma}\right)\mathrm{erf}[\overline{y}(x)/\sqrt{2}]. \tag{7.8}$$

Write (7.8) as $\underbrace{\frac{d\overline{y}}{dx}\frac{\partial}{\partial\overline{y}}}(\frac{1}{2\sigma})\mathrm{erf}[\overline{y}(x)/\sqrt{2}]\frac{1}{\sqrt{2}}$.

$\overline{y} = \left[\left(\frac{\Delta p}{\Delta\beta} - \frac{\Delta\alpha}{\Delta\beta}\right) - \mu\right]/\sigma \Rightarrow$

$\frac{d\overline{y}}{dx} = \frac{1}{\sigma\Delta\beta}$. (7.8)is therefore

$$\frac{1}{\sigma^2\Delta\beta\sqrt{2\pi}}\exp\left[\left(\frac{\Delta p}{\Delta\beta} - \frac{\Delta\alpha}{\Delta\beta}\right) - \mu\right]^2/\sigma^2.$$

Q.E.D.

8. Appendix B: tastes are distributed as in a Γ distribution

The system we have now to solve is

$$\begin{aligned} \Delta p &= \frac{N - \left(\int_0^{\frac{\Delta p - \Delta\alpha}{\Delta\beta}} g(t)dt\right)}{\frac{\partial}{\partial\Delta p}\left(N - \int_0^{\frac{\Delta p-\Delta\alpha}{\Delta\beta}} g(t)dt\right)} \qquad (8.1) \\ g(t) &= \Gamma(\tau). \end{aligned}$$

The Gamma distribution is a continuous random variable defined over the positive interval $[0, +\infty]$. The distribution function is

$$\frac{1}{\Gamma(\tau)\xi^{x}} x^{\tau-1} e^{-\frac{x}{\xi}}.$$

Substituting the expression of the Γ function we have

$$\frac{1}{\left(\int_0^{\infty} x^{\tau-1} e^{-x} dx\right) \xi^{x}} x^{\tau-1} e^{-\frac{x}{\xi}}.$$

When $\xi, \tau \in Z_+$, it becomes

$$\frac{x^{\tau-1} e^{-\frac{x}{\xi}}}{(\tau-1)!\xi^{\tau}}.$$

Solving the system (8.1) out for Δp we find

$$\Delta p - \frac{N - \int_0^{\frac{\Delta p - \Delta\alpha}{\Delta\beta}} \frac{x^{\tau-1} e^{-\frac{x}{\xi}}}{\Gamma(\tau)\xi^{\tau}} dx}{\frac{\partial}{\partial \Delta p} \int_0^{\frac{\Delta p - \Delta\alpha}{\Delta\beta}} \frac{1}{\Gamma(\tau)\xi^{\tau}} x^{\tau-1} e^{-\frac{x}{\xi}} dx} = 0 \tag{8.2}$$

The derivative of the denominator of equation (8.2) is

$$\begin{aligned}
& \frac{1}{\Gamma(\tau)\xi^{\tau}} \lim_{h\to 0} \left(\frac{\int_0^{\frac{\Delta p + h - \Delta\alpha}{\Delta\beta}} x^{\tau-1} e^{-\frac{x}{\xi}} dx - \int_0^{\frac{\Delta p - \Delta\alpha}{\Delta\beta}} x^{\tau-1} e^{-\frac{x}{\xi}} dx}{h} \right) \\
= \; & \frac{1}{\Gamma(\tau)\xi^{\tau}} \left(\int_{\frac{\Delta p - \Delta\alpha}{\Delta\beta}}^{\frac{\Delta p + h - \Delta\alpha}{\Delta\beta}} \frac{x^{\tau-1} e^{-\frac{x}{\xi}}}{h} dx \right) = \\
= \; & \frac{1}{\Gamma(\tau)\xi^{\tau}} \frac{\frac{h}{\Delta\beta} \left(\frac{\Delta p - \Delta\alpha}{\Delta\beta}\right)^{\tau-1} e^{-\frac{\Delta p - \Delta\alpha}{\xi\Delta\beta}}}{h} = & (8.3) \\
= \; & \frac{1}{\Gamma(\tau)\xi^{\tau}} \frac{(\Delta p - \Delta\alpha)^{\tau-1}}{\Delta\beta^{\tau}} e^{-\frac{\Delta p - \Delta\alpha}{\xi\Delta\beta}} & (8.4)
\end{aligned}$$

where the underlined ratio in the second row above derives from $\frac{\Delta p+h-\Delta\alpha}{\Delta\beta} - \frac{\Delta p-\Delta\alpha}{\Delta\beta} = \frac{h}{\Delta\beta}$.

The system (8.1) becomes

$$\Delta p - \frac{N - \int_0^{\frac{\Delta p-\Delta\alpha}{\Delta\beta}} \frac{x^{\tau-1}e^{-\frac{x}{\xi}}}{\Gamma(\tau)\xi^\tau}dx}{\frac{1}{\Gamma(\tau)\xi^\tau}\frac{(\Delta p-\Delta\alpha)^{\tau-1}}{\Delta\beta^\tau}e^{-\frac{\Delta p-\Delta\alpha}{\xi\Delta\beta}}}.$$

Since we are interesting in having $\int_0^\infty \frac{1}{(\int_0^\infty x^{\tau-1}e^{-x}dx)\xi^x}x^{\tau-1}e^{-\frac{x}{\xi}}dx = N$ we get

$$\Delta p - \frac{N - \int_0^{\frac{\Delta p-\Delta\alpha}{\Delta\beta}} \frac{x^{\tau-1}e^{-\frac{x}{\xi}}}{\Gamma(\tau)\xi^\tau}dx * N}{\frac{1}{\Gamma(\tau)\xi^\tau}\frac{(\Delta p-\Delta\alpha)^{\tau-1}}{\Delta\beta^\tau}e^{-\frac{\Delta p-\Delta\alpha}{\xi\Delta\beta}} * N}$$

$$= \Delta p - \frac{\Gamma(\tau)\,\xi^\tau - \int_0^{\frac{\Delta p-\Delta\alpha}{\Delta\beta}} \frac{x^{\tau-1}e^{-\frac{x}{\xi}}}{\Gamma(\tau)\xi^\tau}dx}{\frac{(\Delta p-\Delta\alpha)^{\tau-1}}{\Delta\beta^\tau}e^{-\frac{\Delta p-\Delta\alpha}{\xi\Delta\beta}}}. \tag{8.5}$$

(8.5) is the function to be solved out for Δp. Q.E.D.

9 Appendix C

This Appendix reports parts of the results of the numerical simulations. In each table the optimal price gap, the threshold T_1 ($T1$ in the tables) and the portion of customers choosing the lower quality are indicated for each value of the changing parameters. Parameters kept constant are given in the first row of the tables. The first column contains the parameter under scrutiny. The indexes '_i' (i=1,2,3.) at the third row have been added in order to make information as compact as possible. Results obtained with the normal distribution of tastes can be found from Table 1 to Table 13. Tables 14 to 17 are referred to the gamma distribution of tastes.

Table 1

μ=2	σ=0.5								
	Δβ=0.5			Δβ=1			Δβ=1.5		
Δα	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.2	0.910	1.420	0.123	1.638	1.438	0.131	2.367	1.4447	0.133
0.2267	0.934	1.415	0.121	1.662	1.4354	0.129	2.391	1.4429	0.133
0.2533	0.959	1.411	0.119	1.686	1.4329	0.128	2.415	1.4411	0.132
0.28	0.983	1.406	0.117	1.710	1.4304	0.127	2.439	1.4393	0.131
0.3067	1.008	1.402	0.116	1.735	1.4279	0.126	2.4631	1.4376	0.130
0.3333	1.032	1.398	0.114	1.759	1.4255	0.125	2.4871	1.4359	0.130
0.36	1.057	1.394	0.113	1.783	1.4231	0.124	2.5112	1.4341	0.129
0.3867	1.082	1.390	0.111	1.807	1.4207	0.123	2.5354	1.4325	0.128
0.4133	1.106	1.386	0.110	1.832	1.4184	0.122	2.5595	1.4308	0.127
0.44	1.131	1.382	0.108	1.856	1.4161	0.121	2.5837	1.4291	0.127
0.4667	1.156	1.378	0.107	1.881	1.4138	0.121	2.6079	1.4275	0.126
0.4933	1.181	1.375	0.106	1.905	1.4116	0.120	2.6322	1.4259	0.125
0.52	1.206	1.371	0.104	1.929	1.4093	0.119	2.6564	1.4243	0.125
0.5467	1.231	1.368	0.103	1.954	1.4071	0.118	2.6807	1.4227	0.124
0.5733	1.256	1.365	0.102	1.978	1.405	0.117	2.705	1.4211	0.123
0.6	1.281	1.361	0.101	2.003	1.4029	0.116	2.7293	1.4195	0.123
0.6267	1.306	1.358	0.100	2.027	1.4007	0.115	2.7536	1.4179	0.122
0.6533	1.331	1.355	0.098	2.052	1.3987	0.115	2.778	1.4165	0.122
0.68	1.356	1.352	0.097	2.077	1.3966	0.114	2.8024	1.4149	0.121
0.7067	1.381	1.349	0.096	2.101	1.3946	0.113	2.8268	1.4134	0.120
0.7333	1.406	1.346	0.095	2.126	1.3926	0.112	2.8512	1.4119	0.120
0.76	1.431	1.343	0.094	2.151	1.3906	0.111	2.8757	1.4105	0.119
0.7867	1.457	1.340	0.093	2.175	1.3887	0.111	2.9001	1.4089	0.119
0.8133	1.482	1.337	0.092	2.200	1.3868	0.110	2.9246	1.4075	0.118
0.84	1.507	1.334	0.091	2.225	1.3849	0.109	2.9491	1.4061	0.117
0.8667	1.532	1.331	0.091	2.250	1.3829	0.109	2.9736	1.4046	0.117
0.8933	1.558	1.329	0.090	2.274	1.3811	0.108	2.9982	1.4033	0.116
0.92	1.583	1.326	0.089	2.299	1.3793	0.107	3.0227	1.4018	0.116
0.9467	1.609	1.324	0.088	2.324	1.3774	0.107	3.0473	1.4004	0.115
0.9733	1.634	1.321	0.087	2.349	1.3757	0.106	3.0719	1.3991	0.115

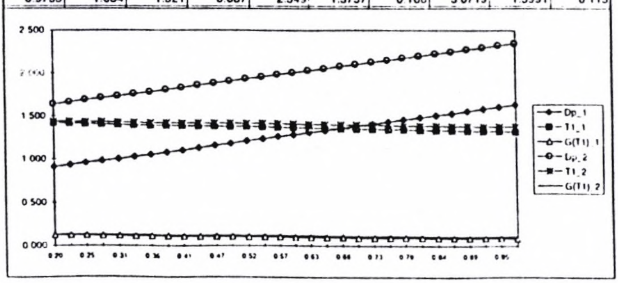

Table 2

Δβ=1	σ=0.5								
	μ=1.5			μ=2.2			μ=2.8		
Δα	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.2	1.196	0.996	0.157	1.820	1.620	0.123	2.374	2.174	0.105
0.2267	1.219	0.992	0.155	1.844	1.617	0.122	2.399	2.172	0.105
0.2533	1.242	0.989	0.153	1.868	1.615	0.121	2.424	2.170	0.104
0.28	1.265	0.985	0.152	1.893	1.613	0.120	2.449	2.169	0.103
0.3067	1.289	0.982	0.150	1.917	1.610	0.119	2.474	2.167	0.103
0.3333	1.312	0.979	0.149	1.942	1.608	0.118	2.499	2.165	0.102
0.36	1.335	0.975	0.147	1.966	1.606	0.117	2.524	2.164	0.102
0.3867	1.359	0.972	0.146	1.991	1.604	0.117	2.549	2.162	0.101
0.4133	1.382	0.969	0.144	2.015	1.602	0.116	2.574	2.160	0.100
0.44	1.406	0.966	0.143	2.040	1.600	0.115	2.599	2.159	0.100
0.4667	1.430	0.963	0.141	2.064	1.598	0.114	2.624	2.157	0.099
0.4933	1.453	0.960	0.140	2.089	1.596	0.113	2.649	2.156	0.099
0.52	1.477	0.957	0.139	2.114	1.594	0.113	2.674	2.154	0.098
0.5467	1.501	0.954	0.138	2.138	1.592	0.112	2.699	2.152	0.098
0.5733	1.525	0.952	0.136	2.163	1.590	0.111	2.724	2.151	0.097
0.6	1.549	0.949	0.135	2.188	1.588	0.110	2.749	2.149	0.097
0.6267	1.573	0.946	0.134	2.213	1.586	0.110	2.775	2.148	0.096
0.6533	1.597	0.943	0.133	2.237	1.584	0.109	2.800	2.146	0.096
0.68	1.621	0.941	0.132	2.262	1.582	0.108	2.825	2.145	0.095
0.7067	1.645	0.938	0.130	2.287	1.580	0.108	2.850	2.143	0.095
0.7333	1.669	0.935	0.129	2.312	1.578	0.107	2.875	2.142	0.094
0.76	1.693	0.933	0.128	2.337	1.577	0.106	2.901	2.141	0.094
0.7867	1.717	0.930	0.127	2.361	1.575	0.106	2.926	2.139	0.093
0.8133	1.741	0.928	0.126	2.386	1.573	0.105	2.951	2.138	0.093
0.84	1.765	0.925	0.125	2.411	1.571	0.104	2.976	2.136	0.092
0.8667	1.790	0.923	0.124	2.436	1.570	0.104	3.002	2.135	0.092
0.8933	1.814	0.921	0.123	2.461	1.568	0.103	3.027	2.134	0.091
0.92	1.838	0.918	0.122	2.486	1.566	0.102	3.052	2.132	0.091
0.9467	1.863	0.916	0.121	2.511	1.564	0.102	3.078	2.131	0.090
0.9733	1.887	0.914	0.121	2.536	1.563	0.101	3.103	2.130	0.090

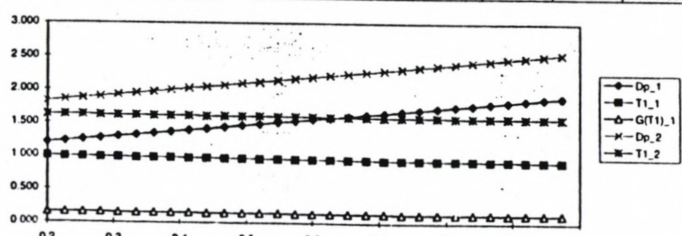

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Table 3

Δβ=1	μ=1.5	σ=0.5			Δβ=1	μ=2.2	σ=0.5		Δβ=1	μ=2.8	σ=0.5	
Δα	Δp_1	T1_1	G(T1)_1	HN_1	Δp_2	T1_2	G(T1)_2	HN_2	Δp_3	T1_3	G(T1)_3	HN_3
0.2	1.196	0.996	0.236	0.764	1.820	1.620	0.203	0.797	2.374	2.174	0.186	0.814
0.216	1.210	0.994	0.235	0.765	1.834	1.618	0.203	0.797	2.389	2.173	0.185	0.815
0.232	1.223	0.991	0.234	0.766	1.849	1.617	0.202	0.798	2.404	2.172	0.185	0.815
0.248	1.237	0.989	0.233	0.767	1.863	1.615	0.202	0.798	2.419	2.171	0.184	0.816
0.264	1.251	0.987	0.232	0.768	1.878	1.614	0.201	0.799	2.434	2.170	0.184	0.816
0.28	1.265	0.985	0.231	0.769	1.893	1.613	0.201	0.799	2.449	2.169	0.184	0.816
0.296	1.279	0.983	0.230	0.770	1.907	1.611	0.200	0.800	2.464	2.168	0.183	0.817
0.312	1.293	0.981	0.229	0.771	1.922	1.610	0.200	0.800	2.479	2.167	0.183	0.817
0.328	1.307	0.979	0.228	0.772	1.937	1.609	0.199	0.801	2.494	2.166	0.182	0.818
0.344	1.321	0.977	0.228	0.772	1.951	1.607	0.199	0.801	2.509	2.165	0.182	0.818
0.36	1.335	0.975	0.227	0.773	1.966	1.606	0.198	0.802	2.524	2.164	0.182	0.818
0.376	1.350	0.974	0.226	0.774	1.981	1.605	0.198	0.802	2.539	2.163	0.181	0.819
0.392	1.364	0.972	0.225	0.775	1.996	1.604	0.197	0.803	2.554	2.162	0.181	0.819
0.408	1.378	0.970	0.224	0.776	2.010	1.602	0.197	0.803	2.569	2.161	0.181	0.819
0.424	1.392	0.968	0.224	0.776	2.025	1.601	0.196	0.804	2.584	2.160	0.180	0.820
0.44	1.406	0.966	0.223	0.777	2.040	1.600	0.196	0.804	2.599	2.159	0.180	0.820
0.456	1.420	0.964	0.222	0.778	2.055	1.599	0.195	0.805	2.614	2.158	0.179	0.821
0.472	1.434	0.962	0.221	0.779	2.069	1.597	0.195	0.805	2.629	2.157	0.179	0.821
0.488	1.449	0.961	0.220	0.780	2.084	1.596	0.194	0.806	2.644	2.156	0.179	0.821
0.504	1.463	0.959	0.220	0.780	2.099	1.595	0.194	0.806	2.659	2.155	0.178	0.822
0.52	1.477	0.957	0.219	0.781	2.114	1.594	0.193	0.807	2.674	2.154	0.178	0.822
0.536	1.491	0.955	0.218	0.782	2.128	1.592	0.193	0.807	2.689	2.153	0.178	0.822
0.552	1.506	0.954	0.218	0.782	2.143	1.591	0.192	0.808	2.704	2.152	0.177	0.823
0.568	1.520	0.952	0.217	0.783	2.158	1.590	0.192	0.808	2.719	2.151	0.177	0.823
0.584	1.534	0.950	0.216	0.784	2.173	1.589	0.191	0.809	2.734	2.150	0.177	0.823
0.6	1.549	0.949	0.215	0.785	2.188	1.588	0.191	0.809	2.749	2.149	0.176	0.824
0.616	1.563	0.947	0.215	0.785	2.203	1.587	0.190	0.810	2.765	2.149	0.176	0.824
0.632	1.577	0.945	0.214	0.786	2.217	1.585	0.190	0.810	2.780	2.148	0.176	0.824
0.648	1.592	0.944	0.213	0.787	2.232	1.584	0.190	0.810	2.795	2.147	0.175	0.825
0.664	1.606	0.942	0.213	0.787	2.247	1.583	0.189	0.811	2.810	2.146	0.175	0.825
0.68	1.621	0.941	0.212	0.788	2.262	1.582	0.189	0.811	2.825	2.145	0.175	0.825

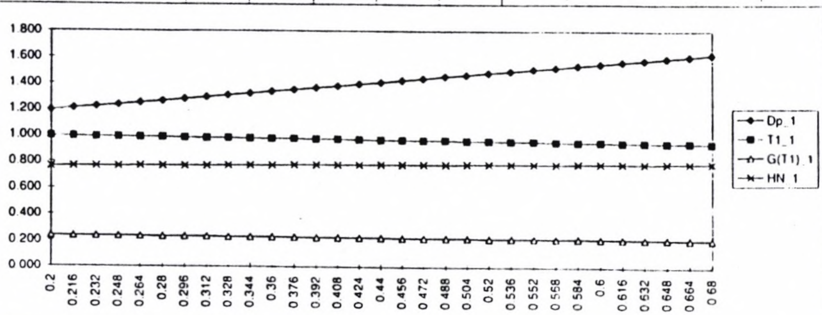


Table 4

$\Delta\beta=1$	$\mu=2$, $\sigma=0.4$			$\sigma=0.6$			$\sigma=0.7$		
$\Delta\alpha$	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.2	1.6777	1.4777	0.0958	1.638	1.438	0.1745	1.6386	1.4386	0.2113
0.2267	1.7025	1.4758	0.0950	1.6621	1.4354	0.1734	1.661	1.4343	0.2095
0.2533	1.7274	1.4741	0.0943	1.6862	1.4329	0.1723	1.6834	1.4301	0.2078
0.28	1.7522	1.4722	0.0935	1.7104	1.4304	0.1712	1.7058	1.4258	0.2060
0.3067	1.7771	1.4704	0.0928	1.7346	1.4279	0.1702	1.7284	1.4217	0.2044
0.3333	1.8021	1.4688	0.0921	1.7588	1.4255	0.1692	1.7509	1.4176	0.2027
0.36	1.827	1.467	0.0913	1.7831	1.4231	0.1682	1.7736	1.4136	0.2011
0.3867	1.852	1.4653	0.0907	1.8074	1.4207	0.1671	1.7963	1.4096	0.1995
0.4133	1.877	1.4637	0.0900	1.8317	1.4184	0.1662	1.819	1.4057	0.1979
0.44	1.902	1.462	0.0893	1.8561	1.4161	0.1652	1.8418	1.4018	0.1964
0.4667	1.927	1.4603	0.0886	1.8805	1.4138	0.1643	1.8647	1.398	0.1949
0.4933	1.9521	1.4588	0.0880	1.9049	1.4116	0.1634	1.8876	1.3943	0.1934
0.52	1.9772	1.4572	0.0874	1.9293	1.4093	0.1624	1.9105	1.3905	0.1920
0.5467	2.0023	1.4556	0.0868	1.9538	1.4071	0.1615	1.9335	1.3868	0.1905
0.5733	2.0274	1.4541	0.0862	1.9783	1.405	0.1607	1.9565	1.3832	0.1891
0.6	2.0525	1.4525	0.0855	2.0029	1.4029	0.1598	1.9796	1.3796	0.1877
0.6267	2.0777	1.451	0.0850	2.0274	1.4007	0.1589	2.0028	1.3761	0.1864
0.6533	2.1029	1.4496	0.0844	2.052	1.3987	0.1581	2.0259	1.3726	0.1851
0.68	2.1281	1.4481	0.0838	2.0766	1.3966	0.1573	2.0492	1.3692	0.1838
0.7067	2.1533	1.4466	0.0833	2.1013	1.3946	0.1565	2.0724	1.3657	0.1824
0.7333	2.1785	1.4452	0.0827	2.1259	1.3926	0.1557	2.0957	1.3624	0.1812
0.76	2.2038	1.4438	0.0822	2.1506	1.3906	0.1549	2.1191	1.3591	0.1799
0.7867	2.229	1.4423	0.0816	2.1754	1.3887	0.1541	2.1424	1.3557	0.1787
0.8133	2.2543	1.441	0.0811	2.2001	1.3868	0.1534	2.1659	1.3526	0.1775
0.84	2.2796	1.4396	0.0806	2.2249	1.3849	0.1526	2.1893	1.3493	0.1763
0.8667	2.3049	1.4382	0.0801	2.2496	1.3829	0.1519	2.2128	1.3461	0.1751
0.8933	2.3303	1.437	0.0796	2.2744	1.3811	0.1512	2.2363	1.343	0.1740
0.92	2.3556	1.4356	0.0791	2.2993	1.3793	0.1505	2.2599	1.3399	0.1728
0.9467	2.381	1.4343	0.0786	2.3241	1.3774	0.1497	2.2835	1.3368	0.1717
0.9733	2.4063	1.433	0.0782	2.349	1.3757	0.149054	2.3071	1.3338	0.1706

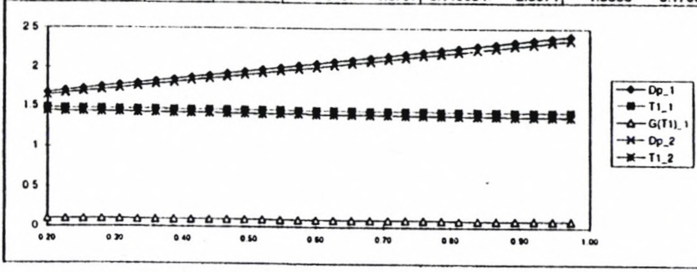


Table 5

σ=0.5	μ=2								
	Δα=0.5			Δα=1			Δα=1.5		
Δβ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.5	1.187	1.374	0.105	1.659	2.319	0.738	2.139	3.278	0.995
0.5333	1.235	1.378	0.107	1.706	2.262	0.700	2.186	3.161	0.990
0.5667	1.283	1.382	0.108	1.754	2.212	0.664	2.232	3.056	0.983
0.6	1.331	1.385	0.109	1.801	2.168	0.632	2.278	2.964	0.973
0.6333	1.379	1.389	0.111	1.848	2.129	0.602	2.325	2.882	0.961
0.6667	1.428	1.391	0.112	1.896	2.094	0.574	2.372	2.807	0.947
0.7	1.476	1.394	0.113	1.943	2.062	0.549	2.419	2.741	0.931
0.7333	1.524	1.397	0.114	1.991	2.033	0.527	2.466	2.680	0.913
0.7667	1.572	1.399	0.115	2.039	2.007	0.506	2.513	2.625	0.894
0.8	1.621	1.401	0.115	2.087	1.983	0.487	2.560	2.575	0.875
0.8333	1.669	1.403	0.116	2.134	1.961	0.469	2.607	2.528	0.855
0.8667	1.717	1.405	0.117	2.182	1.941	0.453	2.654	2.485	0.834
0.9	1.766	1.406	0.118	2.230	1.922	0.438	2.702	2.446	0.814
0.9333	1.814	1.408	0.118	2.278	1.905	0.425	2.749	2.410	0.794
0.9667	1.863	1.410	0.119	2.326	1.889	0.412	2.796	2.375	0.774
1	1.911	1.411	0.119	2.374	1.874	0.400	2.844	2.344	0.754
1.0333	1.959	1.412	0.120	2.422	1.860	0.390	2.891	2.314	0.735
1.0667	2.008	1.414	0.120	2.470	1.847	0.380	2.939	2.286	0.717
1.1	2.056	1.415	0.121	2.518	1.835	0.370	2.987	2.261	0.699
1.1333	2.105	1.416	0.121	2.566	1.823	0.362	3.034	2.236	0.682
1.1667	2.153	1.417	0.122	2.614	1.812	0.354	3.082	2.213	0.665
1.2	2.202	1.418	0.122	2.662	1.802	0.346	3.130	2.191	0.649
1.2333	2.250	1.419	0.123	2.711	1.792	0.339	3.178	2.171	0.634
1.2667	2.299	1.420	0.123	2.759	1.783	0.332	3.225	2.151	0.619
1.3	2.347	1.421	0.123	2.807	1.775	0.326	3.273	2.133	0.605
1.3333	2.396	1.422	0.124	2.855	1.766	0.320	3.321	2.116	0.592
1.3667	2.444	1.423	0.124	2.903	1.759	0.315	3.369	2.099	0.579
1.4	2.493	1.423	0.124	2.952	1.751	0.309	3.417	2.084	0.566
1.4333	2.541	1.424	0.125	3.000	1.744	0.304	3.465	2.069	0.554
1.4667	2.590	1.425	0.125	3.048	1.737	0.300	3.513	2.054	0.543

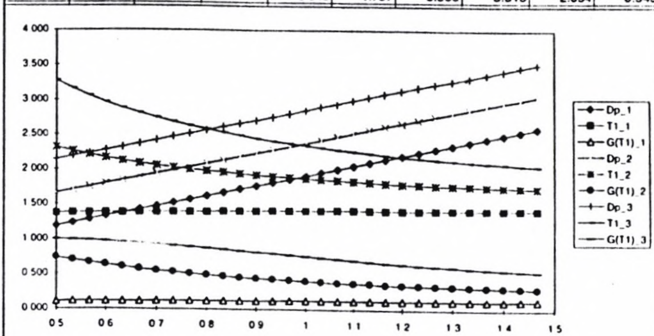


Table 6

σ=0.5	Δα=0.5								
	μ=2.5			μ=3			μ=3.5		
Δβ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.5	1.4219	1.8438	0.0947	1.6593	2.3186	0.0865	1.8986	2.7972	0.0799
0.5333	1.4852	1.8474	0.0959	1.7382	2.3218	0.0875	1.9932	2.7999	0.0807
0.5667	1.5485	1.8502	0.0969	1.8171	2.3242	0.0882	2.0878	2.8018	0.0813
0.6	1.6119	1.8532	0.0979	1.8960	2.3267	0.0890	2.1824	2.8040	0.0820
0.6333	1.6753	1.8558	0.0988	1.9749	2.3289	0.0898	2.2770	2.8059	0.0825
0.6667	1.7387	1.8580	0.0996	2.0538	2.3306	0.0903	2.3717	2.8074	0.0830
0.7	1.8022	1.8603	0.1004	2.1328	2.3326	0.0910	2.4664	2.8091	0.0835
0.7333	1.8656	1.8623	0.1011	2.2117	2.3342	0.0915	2.5610	2.8106	0.0840
0.7667	1.9291	1.8640	0.1017	2.2907	2.3356	0.0920	2.6557	2.8117	0.0843
0.8	1.9926	1.8658	0.1023	2.3697	2.3371	0.0925	2.7504	2.8130	0.0847
0.8333	2.0561	1.8674	0.1029	2.4487	2.3385	0.0929	2.8451	2.8142	0.0851
0.8667	2.1197	1.8688	0.1034	2.5277	2.3396	0.0933	2.9398	2.8150	0.0854
0.9	2.1832	1.8702	0.1039	2.6067	2.3408	0.0937	3.0345	2.8161	0.0857
0.9333	2.2467	1.8715	0.1044	2.6857	2.3419	0.0941	3.1292	2.8171	0.0860
0.9667	2.3103	1.8727	0.1048	2.7648	2.3428	0.0944	3.2239	2.8177	0.0862
1	2.3739	1.8739	0.1052	2.8438	2.3438	0.0947	3.3187	2.8187	0.0865
1.0333	2.4374	1.8750	0.1056	2.9228	2.3447	0.0950	3.4134	2.8195	0.0868
1.0667	2.5010	1.8759	0.1060	3.0019	2.3455	0.0953	3.5081	2.8200	0.0869
1.1	2.5646	1.8769	0.1063	3.0809	2.3463	0.0955	3.6029	2.8208	0.0872
1.1333	2.6282	1.8779	0.1067	3.1600	2.3471	0.0958	3.6976	2.8215	0.0874
1.1667	2.6918	1.8786	0.1070	3.2390	2.3476	0.0960	3.7923	2.8219	0.0875
1.2	2.7554	1.8795	0.1073	3.3181	2.3484	0.0963	3.8871	2.8226	0.0877
1.2333	2.8190	1.8803	0.1076	3.3972	2.3491	0.0965	3.9818	2.8232	0.0879
1.2667	2.8826	1.8810	0.1078	3.4762	2.3496	0.0967	4.0766	2.8236	0.0880
1.3	2.9462	1.8817	0.1081	3.5553	2.3502	0.0969	4.1713	2.8241	0.0882
1.3333	3.0098	1.8824	0.1084	3.6344	2.3509	0.0971	4.2661	2.8246	0.0884
1.3667	3.0735	1.8830	0.1086	3.7134	2.3512	0.0972	4.3608	2.8249	0.0885
1.4	3.1371	1.8836	0.1088	3.7925	2.3518	0.0974	4.4556	2.8254	0.0886
1.4333	3.2007	1.8843	0.1091	3.8716	2.3523	0.0976	4.5504	2.8259	0.0888
1.4667	3.2643	1.8847	0.1092	3.9507	2.3527	0.0977	4.6451	2.8261	0.0889

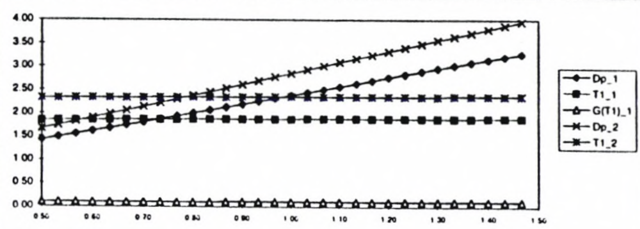


Table 7

	$\Delta\alpha=0.5$	$\mu=3$							
	$\sigma=0.6$			$\sigma=0.8$			$\sigma=1.1$		
$\Delta\beta$	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.5	1.634	2.267	0.111	1.614	2.227	0.167	1.619	2.239	0.244
0.5333	1.711	2.271	0.112	1.691	2.233	0.169	1.698	2.246	0.247
0.5667	1.789	2.274	0.113	1.769	2.239	0.171	1.777	2.253	0.248
0.6	1.867	2.278	0.114	1.846	2.244	0.172	1.855	2.259	0.250
0.6333	1.944	2.281	0.115	1.924	2.248	0.174	1.934	2.264	0.252
0.6667	2.022	2.283	0.116	2.002	2.252	0.175	2.013	2.269	0.253
0.7	2.100	2.286	0.117	2.079	2.256	0.176	2.092	2.274	0.255
0.7333	2.178	2.288	0.118	2.157	2.260	0.177	2.171	2.278	0.256
0.7667	2.256	2.290	0.118	2.235	2.263	0.178	2.249	2.282	0.257
0.8	2.333	2.292	0.119	2.313	2.266	0.179	2.328	2.285	0.258
0.8333	2.411	2.293	0.119	2.390	2.269	0.180	2.407	2.289	0.259
0.8667	2.489	2.295	0.120	2.468	2.271	0.181	2.486	2.292	0.260
0.9	2.567	2.296	0.120	2.546	2.273	0.182	2.565	2.295	0.261
0.9333	2.645	2.298	0.121	2.624	2.276	0.183	2.644	2.298	0.262
0.9667	2.723	2.299	0.121	2.702	2.278	0.183	2.723	2.300	0.262
1	2.800	2.300	0.122	2.780	2.280	0.184	2.803	2.303	0.263
1.0333	2.878	2.302	0.122	2.858	2.282	0.185	2.882	2.305	0.264
1.0667	2.956	2.303	0.123	2.935	2.283	0.185	2.961	2.307	0.264
1.1	3.034	2.304	0.123	3.013	2.285	0.186	3.040	2.309	0.265
1.1333	3.112	2.305	0.123	3.091	2.287	0.186	3.119	2.311	0.265
1.1667	3.190	2.306	0.124	3.169	2.288	0.187	3.198	2.313	0.266
1.2	3.268	2.307	0.124	3.247	2.289	0.187	3.277	2.314	0.267
1.2333	3.346	2.307	0.124	3.325	2.291	0.188	3.356	2.316	0.267
1.2667	3.424	2.308	0.124	3.403	2.292	0.188	3.435	2.317	0.267
1.3	3.502	2.309	0.125	3.481	2.293	0.188	3.515	2.319	0.268
1.3333	3.580	2.310	0.125	3.559	2.294	0.189	3.594	2.320	0.268
1.3667	3.658	2.310	0.125	3.637	2.295	0.189	3.673	2.322	0.269
1.4	3.735	2.311	0.125	3.715	2.296	0.190	3.752	2.323	0.269
1.4333	3.813	2.312	0.126	3.793	2.297	0.190	3.831	2.324	0.269
1.4667	3.891	2.312	0.126	3.871	2.298	0.190	3.910	2.325	0.270

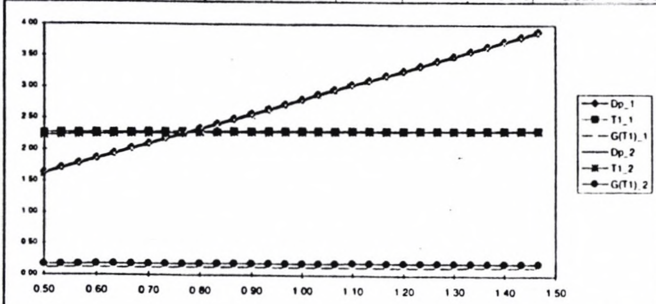
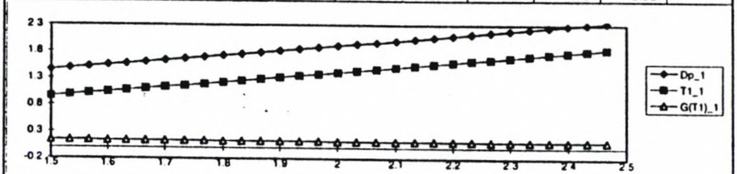


Table 8

σ=0.5	Δβ=1								
	Δα=0.5			Δα=1			Δα=1.5		
μ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
1.5	1.459	0.959	0.140	1.912	0.912	0.120	2.374	0.874	0.105
1.5333	1.489	0.989	0.138	1.942	0.942	0.118	2.405	0.905	0.105
1.5667	1.519	1.019	0.136	1.973	0.973	0.117	2.437	0.937	0.104
1.6	1.548	1.048	0.135	2.003	1.003	0.116	2.468	0.968	0.103
1.6333	1.578	1.078	0.133	2.034	1.034	0.115	2.499	0.999	0.102
1.6667	1.608	1.108	0.132	2.065	1.065	0.114	2.530	1.030	0.101
1.7	1.638	1.138	0.131	2.095	1.095	0.113	2.561	1.061	0.101
1.7333	1.668	1.168	0.129	2.126	1.126	0.112	2.593	1.093	0.100
1.7667	1.698	1.198	0.128	2.157	1.157	0.111	2.624	1.124	0.099
1.8	1.729	1.229	0.127	2.188	1.188	0.110	2.655	1.155	0.099
1.8333	1.759	1.259	0.125	2.219	1.219	0.109	2.687	1.187	0.098
1.8667	1.789	1.289	0.124	2.250	1.250	0.109	2.718	1.218	0.097
1.9	1.820	1.320	0.123	2.281	1.281	0.108	2.749	1.249	0.097
1.9333	1.850	1.350	0.122	2.312	1.312	0.107	2.781	1.281	0.096
1.9667	1.881	1.381	0.121	2.343	1.343	0.106	2.812	1.312	0.095
2	1.911	1.411	0.119	2.374	1.374	0.105	2.844	1.344	0.095
2.0333	1.942	1.442	0.118	2.405	1.405	0.104	2.875	1.375	0.094
2.0667	1.972	1.472	0.117	2.436	1.436	0.104	2.907	1.407	0.093
2.1	2.003	1.503	0.116	2.467	1.467	0.103	2.938	1.438	0.093
2.1333	2.034	1.534	0.115	2.499	1.499	0.102	2.970	1.470	0.092
2.1667	2.064	1.564	0.114	2.530	1.530	0.101	3.002	1.502	0.092
2.2	2.095	1.595	0.113	2.561	1.561	0.101	3.033	1.533	0.091
2.2333	2.126	1.626	0.112	2.592	1.592	0.100	3.065	1.565	0.091
2.2667	2.157	1.657	0.111	2.624	1.624	0.099	3.097	1.597	0.090
2.3	2.188	1.688	0.110	2.655	1.655	0.099	3.128	1.628	0.090
2.3333	2.219	1.719	0.109	2.687	1.687	0.098	3.160	1.660	0.089
2.3667	2.250	1.750	0.109	2.718	1.718	0.097	3.192	1.692	0.088
2.4	2.281	1.781	0.108	2.749	1.749	0.097	3.223	1.723	0.088
2.4333	2.312	1.812	0.107	2.781	1.781	0.096	3.255	1.755	0.087
2.4667	2.343	1.843	0.106	2.812	1.812	0.095	3.287	1.787	0.087



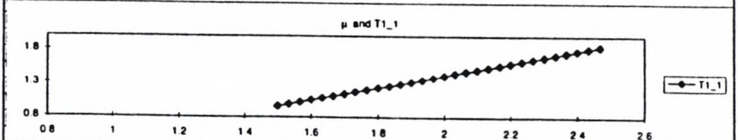


Table 9

σ=0.5		Δα=1							
	Δβ=0.5			Δβ=0.8			Δβ=1.5		
μ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
1.5	1.4222	0.844	0.095	1.7135	0.892	0.112	2.4128	0.942	0.132
1.5333	1.4379	0.876	0.094	1.7382	0.923	0.111	2.4577	0.972	0.131
1.5667	1.4536	0.907	0.094	1.7628	0.954	0.110	2.5027	1.002	0.129
1.6	1.4693	0.939	0.093	1.7875	0.984	0.109	2.5479	1.032	0.128
1.6333	1.4851	0.970	0.092	1.8123	1.015	0.108	2.5931	1.062	0.127
1.6667	1.5009	1.002	0.092	1.8371	1.046	0.107	2.6385	1.092	0.125
1.7	1.5167	1.033	0.091	1.8619	1.077	0.107	2.6839	1.123	0.124
1.7333	1.5325	1.065	0.091	1.8867	1.108	0.106	2.7294	1.153	0.123
1.7667	1.5483	1.097	0.090	1.9116	1.140	0.105	2.7751	1.183	0.122
1.8	1.5641	1.128	0.090	1.9365	1.171	0.104	2.8208	1.214	0.121
1.8333	1.58	1.160	0.089	1.9615	1.202	0.103	2.8665	1.244	0.119
1.8667	1.5958	1.192	0.088	1.9864	1.233	0.103	2.9124	1.275	0.118
1.9	1.6117	1.223	0.088	2.0114	1.264	0.102	2.9583	1.306	0.117
1.9333	1.6276	1.255	0.088	2.0364	1.296	0.101	3.0043	1.336	0.116
1.9667	1.6434	1.287	0.087	2.0614	1.327	0.100	3.0504	1.367	0.115
2	1.6593	1.319	0.086	2.0865	1.358	0.100	3.0965	1.398	0.114
2.0333	1.6752	1.350	0.086	2.1116	1.390	0.099	3.1427	1.428	0.113
2.0667	1.6911	1.382	0.085	2.1367	1.421	0.098	3.1889	1.459	0.112
2.1	1.7071	1.414	0.085	2.1618	1.452	0.098	3.2352	1.490	0.111
2.1333	1.723	1.446	0.085	2.1869	1.484	0.097	3.2816	1.521	0.110
2.1667	1.7389	1.478	0.084	2.2121	1.515	0.096	3.328	1.552	0.109
2.2	1.7548	1.510	0.084	2.2372	1.547	0.096	3.3744	1.583	0.109
2.2333	1.7708	1.542	0.083	2.2624	1.578	0.095	3.421	1.614	0.108
2.2667	1.7867	1.573	0.083	2.2876	1.610	0.094	3.4675	1.645	0.107
2.3	1.8027	1.605	0.082	2.3129	1.641	0.094	3.5141	1.676	0.106
2.3333	1.8187	1.637	0.082	2.3381	1.673	0.093	3.5608	1.707	0.105
2.3667	1.8346	1.669	0.082	2.3634	1.704	0.093	3.6075	1.738	0.104
2.4	1.8506	1.701	0.081	2.3886	1.736	0.092	3.6543	1.770	0.104
2.4333	1.8666	1.733	0.081	2.4139	1.767	0.091	3.7011	1.801	0.103
2.4667	1.8826	1.765	0.080	2.4392	1.799	0.091	3.7479	1.832	0.102

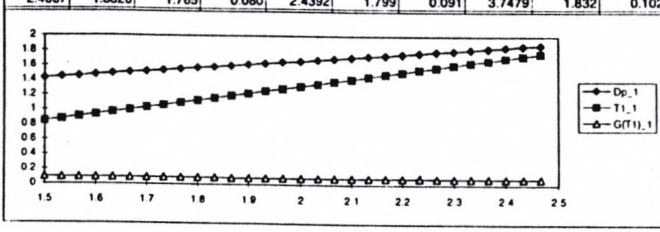


Table 10

$\Delta\alpha=0.5$	$\Delta\beta=1$								
	$\sigma=0.4$			$\sigma=0.6$			$\sigma=0.8$		
μ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
1.5	1.493	0.993	0.102	1.455	0.955	0.182	1.537	1.037	0.282
1.5333	1.523	1.023	0.101	1.483	0.983	0.180	1.562	1.062	0.278
1.5667	1.554	1.054	0.100	1.512	1.012	0.177	1.586	1.086	0.274
1.6	1.585	1.085	0.099	1.540	1.040	0.175	1.611	1.111	0.270
1.6333	1.616	1.116	0.098	1.569	1.069	0.173	1.636	1.136	0.267
1.6667	1.647	1.147	0.097	1.597	1.097	0.171	1.661	1.161	0.264
1.7	1.678	1.178	0.096	1.626	1.126	0.169	1.686	1.186	0.260
1.7333	1.709	1.209	0.095	1.655	1.155	0.168	1.712	1.212	0.257
1.7667	1.740	1.240	0.094	1.684	1.184	0.166	1.738	1.238	0.254
1.8	1.771	1.271	0.093	1.713	1.213	0.164	1.764	1.264	0.251
1.8333	1.802	1.302	0.092	1.742	1.242	0.162	1.790	1.290	0.248
1.8667	1.833	1.333	0.091	1.772	1.272	0.161	1.816	1.316	0.246
1.9	1.865	1.365	0.090	1.801	1.301	0.159	1.843	1.343	0.243
1.9333	1.896	1.396	0.089	1.830	1.330	0.157	1.870	1.370	0.240
1.9667	1.927	1.427	0.089	1.860	1.360	0.156	1.896	1.396	0.238
2	1.958	1.458	0.088	1.889	1.389	0.154	1.924	1.424	0.236
2.0333	1.990	1.490	0.087	1.919	1.419	0.153	1.951	1.451	0.233
2.0667	2.021	1.521	0.086	1.949	1.449	0.152	1.978	1.478	0.231
2.1	2.053	1.553	0.086	1.979	1.479	0.150	2.005	1.505	0.229
2.1333	2.084	1.584	0.085	2.008	1.508	0.149	2.033	1.533	0.227
2.1667	2.116	1.616	0.084	2.038	1.538	0.147	2.061	1.561	0.224
2.2	2.147	1.647	0.083	2.068	1.568	0.146	2.089	1.589	0.222
2.2333	2.179	1.679	0.083	2.098	1.598	0.145	2.116	1.616	0.220
2.2667	2.210	1.710	0.082	2.128	1.628	0.144	2.144	1.644	0.218
2.3	2.242	1.742	0.081	2.159	1.659	0.142	2.173	1.673	0.216
2.3333	2.273	1.773	0.081	2.189	1.689	0.141	2.201	1.701	0.215
2.3667	2.305	1.805	0.080	2.219	1.719	0.140	2.229	1.729	0.213
2.4	2.337	1.837	0.079	2.249	1.749	0.139	2.257	1.757	0.211
2.4333	2.368	1.868	0.079	2.279	1.779	0.138	2.286	1.786	0.209
2.4667	2.400	1.900	0.078	2.310	1.810	0.137	2.314	1.814	0.207

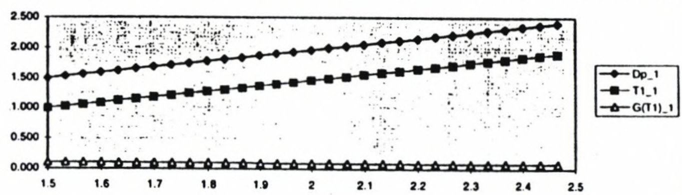


Table 11

σ	Δβ=1 Δα=0.5 Δp_1	μ=2 T1_1	G(T1)_1	Δα=0.7 Δp_2	T1_2	G(T1)_2	Δα=1 Δp_3	T1_3	G(T1)_3
0.4	1.958	1.458	0.088	2.147	1.447	0.083	2.432	1.432	0.078
0.4067	1.954	1.454	0.090	2.143	1.443	0.085	2.427	1.427	0.079
0.4133	1.951	1.451	0.092	2.139	1.439	0.087	2.423	1.423	0.081
0.42	1.947	1.447	0.094	2.135	1.435	0.089	2.418	1.418	0.083
0.4267	1.943	1.443	0.096	2.131	1.431	0.091	2.414	1.414	0.085
0.4333	1.940	1.440	0.098	2.127	1.427	0.093	2.410	1.410	0.087
0.44	1.936	1.436	0.100	2.123	1.423	0.095	2.406	1.406	0.088
0.4467	1.933	1.433	0.102	2.120	1.420	0.097	2.402	1.402	0.090
0.4533	1.930	1.430	0.104	2.116	1.416	0.099	2.398	1.398	0.092
0.46	1.927	1.427	0.106	2.113	1.413	0.101	2.394	1.394	0.094
0.4667	1.924	1.424	0.109	2.110	1.410	0.103	2.390	1.390	0.096
0.4733	1.921	1.421	0.111	2.106	1.406	0.105	2.387	1.387	0.098
0.48	1.918	1.418	0.113	2.103	1.403	0.107	2.384	1.384	0.100
0.4867	1.916	1.416	0.115	2.101	1.401	0.109	2.380	1.380	0.101
0.4933	1.913	1.413	0.117	2.098	1.398	0.111	2.377	1.377	0.103
0.5	1.911	1.411	0.119	2.095	1.395	0.113	2.374	1.374	0.105
0.5067	1.909	1.409	0.122	2.093	1.393	0.115	2.371	1.371	0.107
0.5133	1.907	1.407	0.124	2.090	1.390	0.117	2.368	1.368	0.109
0.52	1.905	1.405	0.126	2.088	1.388	0.120	2.365	1.365	0.111
0.5267	1.903	1.403	0.128	2.086	1.386	0.122	2.363	1.363	0.113
0.5333	1.901	1.401	0.131	2.084	1.384	0.124	2.360	1.360	0.115
0.54	1.899	1.399	0.133	2.082	1.382	0.126	2.358	1.358	0.117
0.5467	1.898	1.398	0.135	2.080	1.380	0.128	2.355	1.355	0.119
0.5533	1.896	1.396	0.138	2.078	1.378	0.130	2.353	1.353	0.121
0.56	1.895	1.395	0.140	2.076	1.376	0.133	2.351	1.351	0.123
0.5667	1.894	1.394	0.142	2.075	1.375	0.135	2.349	1.349	0.125
0.5733	1.893	1.393	0.145	2.073	1.373	0.137	2.347	1.347	0.127
0.58	1.892	1.392	0.147	2.072	1.372	0.139	2.345	1.345	0.129
0.5867	1.891	1.391	0.150	2.071	1.371	0.142	2.343	1.343	0.132
0.5933	1.890	1.390	0.152	2.069	1.369	0.144	2.342	1.342	0.134

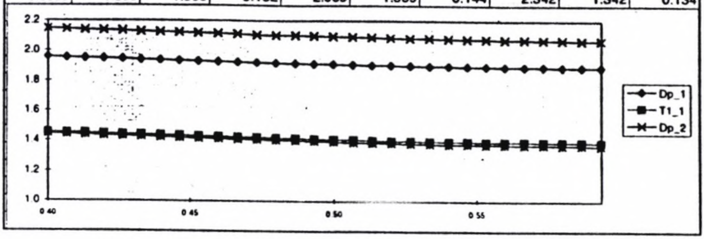


Table 12

Δα=0.5	μ=2								
	Δβ=0.5			Δβ=0.7			Δβ=1.3		
σ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.4	1.216	1.432	0.078	1.512	1.446	0.083	2.405	1.465	0.091
0.4067	1.214	1.427	0.079	1.509	1.442	0.085	2.400	1.462	0.093
0.4133	1.211	1.423	0.081	1.506	1.438	0.087	2.395	1.458	0.095
0.42	1.209	1.418	0.083	1.504	1.434	0.089	2.391	1.454	0.097
0.4267	1.207	1.414	0.085	1.501	1.430	0.091	2.386	1.451	0.099
0.4333	1.205	1.410	0.087	1.498	1.426	0.093	2.382	1.448	0.101
0.44	1.203	1.406	0.088	1.496	1.422	0.095	2.378	1.444	0.103
0.4467	1.201	1.402	0.090	1.493	1.419	0.097	2.374	1.441	0.106
0.4533	1.199	1.398	0.092	1.491	1.415	0.098	2.370	1.438	0.108
0.46	1.197	1.394	0.094	1.488	1.412	0.101	2.366	1.436	0.110
0.4667	1.195	1.390	0.096	1.486	1.409	0.103	2.363	1.433	0.112
0.4733	1.193	1.387	0.098	1.484	1.405	0.105	2.359	1.430	0.114
0.48	1.192	1.383	0.099	1.482	1.402	0.107	2.356	1.428	0.117
0.4867	1.190	1.380	0.101	1.480	1.400	0.109	2.353	1.425	0.119
0.4933	1.189	1.377	0.103	1.478	1.397	0.111	2.350	1.423	0.121
0.5	1.187	1.374	0.105	1.476	1.394	0.113	2.347	1.421	0.123
0.5067	1.185	1.371	0.107	1.474	1.391	0.115	2.345	1.419	0.126
0.5133	1.184	1.368	0.109	1.472	1.389	0.117	2.342	1.417	0.128
0.52	1.183	1.365	0.111	1.471	1.387	0.119	2.340	1.415	0.130
0.5267	1.181	1.363	0.113	1.469	1.384	0.121	2.338	1.413	0.133
0.5333	1.180	1.360	0.115	1.468	1.382	0.123	2.336	1.412	0.135
0.54	1.179	1.358	0.117	1.466	1.380	0.126	2.334	1.410	0.137
0.5467	1.178	1.355	0.119	1.465	1.378	0.128	2.332	1.409	0.140
0.5533	1.177	1.353	0.121	1.464	1.377	0.130	2.330	1.408	0.142
0.56	1.175	1.351	0.123	1.462	1.375	0.132	2.329	1.407	0.145
0.5667	1.174	1.349	0.125	1.461	1.373	0.134	2.328	1.406	0.147
0.5733	1.174	1.347	0.127	1.460	1.372	0.137	2.326	1.405	0.150
0.58	1.173	1.345	0.129	1.459	1.370	0.139	2.325	1.404	0.152
0.5867	1.172	1.343	0.132	1.458	1.369	0.141	2.325	1.403	0.155
0.5933	1.171	1.342	0.134	1.458	1.368	0.143	2.324	1.403	0.157

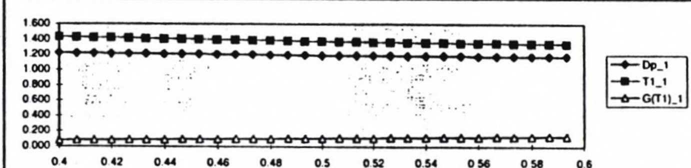


Table 13

$\Delta\alpha=0.5$	$\Delta\beta=1$								
	$\mu=1.5$			$\mu=2.3$			$\mu=2.7$		
σ	Δp_1	T1_1	G(T1)_1	Δp_2	T1_2	G(T1)_2	Δp_3	T1_3	G(T1)_3
0.4	1.493	0.993	0.102	2.242	1.742	0.081	2.623	2.123	0.074
0.4067	1.490	0.990	0.105	2.237	1.737	0.083	2.618	2.118	0.076
0.4133	1.487	0.987	0.107	2.233	1.733	0.085	2.613	2.113	0.078
0.42	1.484	0.984	0.109	2.229	1.729	0.087	2.608	2.108	0.079
0.4267	1.481	0.981	0.112	2.225	1.725	0.089	2.604	2.104	0.081
0.4333	1.478	0.978	0.114	2.221	1.721	0.091	2.599	2.099	0.083
0.44	1.476	0.976	0.117	2.217	1.717	0.093	2.595	2.095	0.085
0.4467	1.473	0.973	0.119	2.213	1.713	0.095	2.591	2.091	0.086
0.4533	1.471	0.971	0.122	2.210	1.710	0.096	2.587	2.087	0.088
0.46	1.469	0.969	0.124	2.206	1.706	0.098	2.583	2.083	0.090
0.4667	1.467	0.967	0.127	2.203	1.703	0.100	2.579	2.079	0.092
0.4733	1.465	0.965	0.129	2.200	1.700	0.102	2.575	2.075	0.093
0.48	1.464	0.964	0.132	2.197	1.697	0.104	2.572	2.072	0.095
0.4867	1.462	0.962	0.135	2.193	1.693	0.106	2.568	2.068	0.097
0.4933	1.461	0.961	0.137	2.191	1.691	0.108	2.565	2.065	0.099
0.5	1.459	0.959	0.140	2.188	1.688	0.110	2.561	2.061	0.101
0.5067	1.458	0.958	0.142	2.185	1.685	0.112	2.558	2.058	0.103
0.5133	1.457	0.957	0.145	2.182	1.682	0.114	2.555	2.055	0.104
0.52	1.456	0.956	0.148	2.180	1.680	0.117	2.552	2.052	0.106
0.5267	1.455	0.955	0.151	2.178	1.678	0.119	2.549	2.049	0.108
0.5333	1.455	0.955	0.153	2.175	1.675	0.121	2.546	2.046	0.110
0.54	1.454	0.954	0.156	2.173	1.673	0.123	2.543	2.043	0.112
0.5467	1.454	0.954	0.159	2.171	1.671	0.125	2.541	2.041	0.114
0.5533	1.454	0.954	0.162	2.169	1.669	0.127	2.538	2.038	0.116
0.56	1.453	0.953	0.164	2.167	1.667	0.129	2.536	2.036	0.118
0.5667	1.453	0.953	0.167	2.166	1.666	0.131	2.533	2.033	0.120
0.5733	1.453	0.953	0.170	2.164	1.664	0.134	2.531	2.031	0.122
0.58	1.454	0.954	0.173	2.162	1.662	0.136	2.529	2.029	0.124
0.5867	1.454	0.954	0.176	2.161	1.661	0.138	2.527	2.027	0.126
0.5933	1.455	0.955	0.179	2.160	1.660	0.140	2.525	2.025	0.128

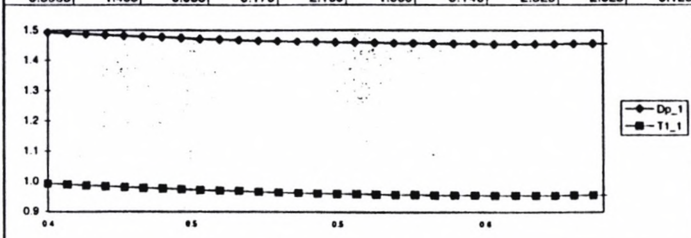


Table 14

Δβ=0.5	τ=2														
	Δα=0.1					Δα=0.5				Δα=1					
ξ	Δρ_1	T1_1	mean	N0+N1_1	NH_1	Δρ_2	T1_2	N0+N1_2	NH_3	Δρ_3	T1_3	N0+N1_3	NH_3		
1	0.8385	1.477	2.000	0.434	0.566	1	1	0.264	0.736	1.309	0.618	0.128	0.872		
1.015	0.8506	1.5012	1.970	0.435	0.565	1.0113	1.0226	0.267	0.733	1.318	0.636	0.131	0.869		
1.03	0.8627	1.5254	1.942	0.436	0.564	1.0226	1.0452	0.270	0.730	1.327	0.653	0.133	0.867		
1.045	0.8748	1.5496	1.914	0.436	0.564	1.0339	1.0678	0.272	0.728	1.336	0.671	0.136	0.864		
1.06	0.8869	1.5738	1.887	0.437	0.563	1.0452	1.0904	0.275	0.725	1.345	0.690	0.139	0.861		
1.075	0.8991	1.5982	1.860	0.438	0.562	1.0566	1.1132	0.277	0.723	1.354	0.708	0.141	0.859		
1.09	0.9112	1.6224	1.835	0.438	0.562	1.068	1.136	0.280	0.720	1.363	0.726	0.144	0.856		
1.105	0.9233	1.6466	1.810	0.439	0.561	1.0794	1.1588	0.282	0.718	1.372	0.745	0.147	0.853		
1.12	0.9354	1.6708	1.786	0.439	0.561	1.0908	1.1816	0.284	0.716	1.382	0.763	0.149	0.851		
1.135	0.9475	1.695	1.762	0.440	0.560	1.1023	1.2046	0.287	0.713	1.391	0.782	0.152	0.848		
1.15	0.9596	1.7192	1.739	0.440	0.560	1.1137	1.2274	0.289	0.711	1.401	0.801	0.155	0.845		
1.165	0.9717	1.7434	1.717	0.441	0.559	1.1252	1.2504	0.291	0.709	1.410	0.820	0.157	0.843		
1.18	0.9838	1.7676	1.695	0.441	0.559	1.1367	1.2734	0.293	0.707	1.420	0.839	0.160	0.840		
1.195	0.996	1.792	1.674	0.442	0.558	1.1482	1.2964	0.295	0.705	1.429	0.858	0.162	0.838		
1.21	1.0081	1.8162	1.653	0.443	0.557	1.1598	1.3196	0.298	0.702	1.439	0.878	0.165	0.835		
1.225	1.0202	1.8404	1.633	0.443	0.557	1.1713	1.3426	0.300	0.700	1.449	0.897	0.167	0.833		
1.24	1.0323	1.8646	1.613	0.443	0.557	1.1829	1.3658	0.302	0.698	1.459	0.917	0.170	0.830		
1.255	1.0444	1.8888	1.594	0.444	0.556	1.1945	1.389	0.303	0.697	1.468	0.937	0.172	0.828		
1.27	1.0565	1.913	1.575	0.444	0.556	1.2061	1.4122	0.305	0.695	1.478	0.956	0.174	0.826		
1.285	1.0687	1.9374	1.556	0.445	0.555	1.2177	1.4354	0.307	0.693	1.488	0.976	0.177	0.823		
1.3	1.0808	1.9616	1.538	0.445	0.555	1.2293	1.4586	0.309	0.691	1.498	0.996	0.179	0.821		
1.315	1.0929	1.9858	1.521	0.446	0.554	1.2409	1.4818	0.311	0.689	1.508	1.016	0.182	0.818		
1.33	1.105	2.01	1.504	0.446	0.554	1.2526	1.5052	0.313	0.687	1.518	1.037	0.184	0.816		
1.345	1.1171	2.0342	1.487	0.446	0.554	1.2643	1.5286	0.314	0.686	1.528	1.057	0.186	0.814		
1.36	1.1293	2.0586	1.471	0.447	0.553	1.2759	1.5518	0.316	0.684	1.539	1.077	0.188	0.812		
1.375	1.1414	2.0828	1.455	0.447	0.553	1.2876	1.5752	0.318	0.682	1.549	1.098	0.191	0.809		
1.39	1.1535	2.107	1.439	0.447	0.553	1.2993	1.5986	0.319	0.681	1.559	1.118	0.193	0.807		
1.405	1.1656	2.1312	1.423	0.448	0.552	1.311	1.622	0.321	0.679	1.569	1.139	0.195	0.805		
1.42	1.1777	2.1554	1.408	0.448	0.552	1.3227	1.6454	0.322	0.678	1.580	1.159	0.197	0.803		
1.435	1.1899	2.1798	1.394	0.449	0.551	1.3344	1.6688	0.324	0.676	1.590	1.180	0.199	0.801		

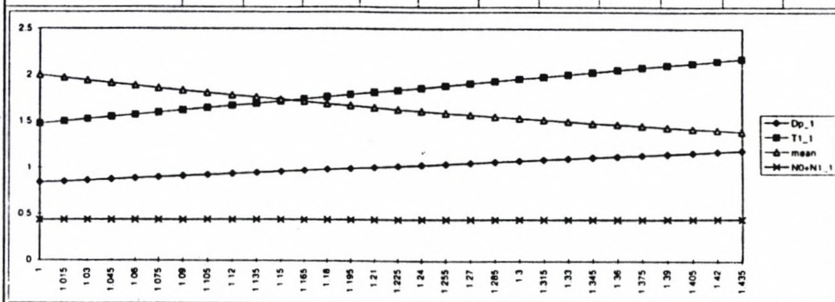


Table 15

Δα=0.5	τ=2											
	Δβ=1				Δβ=1.5				Δβ=2			
ξ	Δp_1	T1_1	N0+N1_1	NH_1	Δp_2	T1_2	N0+N1_2	NH_2	Δp_3	T1_3	N0+N1_3	NH_3
1	1.781	1.281	0.366	0.634	2.581	1.387	0.404	0.596	3.386	1.443	0.423	0.577
1.015	1.805	1.305	0.368	0.632	2.617	1.412	0.405	0.595	3.434	1.467	0.424	0.576
1.03	1.829	1.329	0.370	0.630	2.654	1.436	0.406	0.594	3.483	1.491	0.425	0.575
1.045	1.852	1.352	0.371	0.629	2.690	1.460	0.407	0.593	3.531	1.516	0.425	0.575
1.06	1.876	1.376	0.373	0.627	2.726	1.484	0.408	0.592	3.580	1.540	0.426	0.574
1.075	1.900	1.400	0.374	0.626	2.762	1.508	0.409	0.591	3.628	1.564	0.427	0.573
1.09	1.924	1.424	0.376	0.624	2.798	1.532	0.410	0.590	3.676	1.588	0.428	0.572
1.105	1.948	1.448	0.377	0.623	2.834	1.556	0.411	0.589	3.725	1.612	0.428	0.572
1.12	1.972	1.472	0.378	0.622	2.871	1.580	0.412	0.588	3.773	1.637	0.429	0.571
1.135	1.996	1.496	0.380	0.620	2.907	1.605	0.413	0.587	3.821	1.661	0.430	0.570
1.15	2.020	1.520	0.381	0.619	2.943	1.629	0.414	0.586	3.870	1.685	0.430	0.570
1.165	2.044	1.544	0.382	0.618	2.979	1.653	0.415	0.585	3.918	1.709	0.431	0.569
1.18	2.068	1.568	0.383	0.617	3.016	1.677	0.415	0.585	3.967	1.733	0.432	0.568
1.195	2.092	1.592	0.385	0.615	3.052	1.701	0.416	0.584	4.015	1.758	0.432	0.568
1.21	2.116	1.616	0.386	0.614	3.088	1.725	0.417	0.583	4.064	1.782	0.433	0.567
1.225	2.140	1.640	0.387	0.613	3.124	1.749	0.418	0.582	4.112	1.806	0.434	0.566
1.24	2.164	1.664	0.388	0.612	3.160	1.774	0.419	0.581	4.160	1.830	0.434	0.566
1.255	2.188	1.688	0.389	0.611	3.197	1.798	0.419	0.581	4.209	1.854	0.435	0.565
1.27	2.212	1.712	0.390	0.610	3.233	1.822	0.420	0.580	4.257	1.879	0.435	0.565
1.285	2.236	1.736	0.391	0.609	3.269	1.846	0.421	0.579	4.306	1.903	0.436	0.564
1.3	2.260	1.760	0.392	0.608	3.305	1.870	0.421	0.579	4.354	1.927	0.436	0.564
1.315	2.284	1.784	0.393	0.607	3.342	1.894	0.422	0.578	4.403	1.951	0.437	0.563
1.33	2.308	1.808	0.394	0.606	3.378	1.919	0.423	0.577	4.451	1.975	0.437	0.563
1.345	2.332	1.832	0.395	0.605	3.414	1.943	0.423	0.577	4.499	2.000	0.438	0.562
1.36	2.356	1.856	0.396	0.604	3.451	1.967	0.424	0.576	4.548	2.024	0.438	0.562
1.375	2.380	1.880	0.397	0.603	3.487	1.991	0.425	0.575	4.596	2.048	0.439	0.561
1.39	2.405	1.905	0.398	0.602	3.523	2.015	0.425	0.575	4.645	2.072	0.439	0.561
1.405	2.429	1.929	0.399	0.601	3.559	2.040	0.426	0.574	4.693	2.097	0.440	0.560
1.42	2.453	1.953	0.400	0.600	3.596	2.064	0.426	0.574	4.742	2.121	0.440	0.560

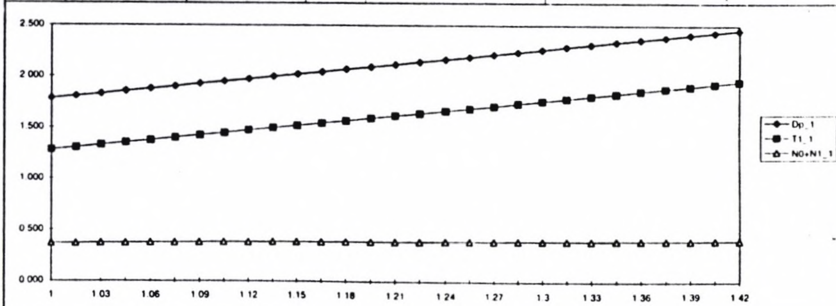


Table 16

	$\Delta\beta$=0.5					ξ=1							
	$\Delta\alpha$=0.1					$\Delta\alpha$=0.5					$\Delta\alpha$=1		
t	Δp_1	T1_1	mean	N0+N1_1	NH_1	Δp_2	T1_2	N0+N1_2	NH_2	T1_3	Δp_3	N0+N1_3	NH_3
2	0.84	1.48	1.333	0.434	0.566	1.00	1.00	0.264	0.736	0.62	1.31	0.128	0.87[illegible]
2.015	0.84	1.49	1.343	0.433	0.567	1.01	1.01	0.265	0.735	0.63	1.32	0.129	0.8[illegible]
2.03	0.85	1.50	1.353	0.432	0.568	1.01	1.02	0.265	0.735	0.64	1.32	0.130	0.8[illegible]
2.045	0.85	1.51	1.363	0.431	0.569	1.02	1.04	0.265	0.735	0.65	1.33	0.131	0.8[illegible]
2.06	0.86	1.52	1.373	0.429	0.571	1.02	1.05	0.265	0.735	0.67	1.33	0.133	0.8[illegible]
2.075	0.86	1.53	1.383	0.428	0.572	1.03	1.06	0.265	0.735	0.68	1.34	0.134	0.866
2.09	0.87	1.54	1.393	0.427	0.573	1.04	1.07	0.265	0.735	0.69	1.35	0.135	0.86[illegible]
2.105	0.87	1.55	1.403	0.426	0.574	1.04	1.08	0.266	0.734	0.70	1.35	0.136	0.864
2.12	0.88	1.56	1.413	0.424	0.576	1.05	1.10	0.266	0.734	0.72	1.36	0.137	0.863
2.135	0.88	1.57	1.423	0.423	0.577	1.05	1.11	0.266	0.734	0.73	1.36	0.138	0.862
2.15	0.89	1.58	1.433	0.422	0.578	1.06	1.12	0.266	0.734	0.74	1.37	0.139	0.861
2.165	0.89	1.59	1.443	0.421	0.579	1.07	1.13	0.266	0.734	0.75	1.38	0.140	0.860
2.18	0.90	1.60	1.453	0.420	0.580	1.07	1.14	0.266	0.734	0.76	1.38	0.141	0.859
2.195	0.90	1.61	1.463	0.419	0.581	1.08	1.15	0.266	0.734	0.78	1.39	0.142	0.858
2.21	0.91	1.62	1.473	0.417	0.583	1.08	1.17	0.266	0.734	0.79	1.39	0.142	0.858
2.225	0.91	1.63	1.483	0.416	0.584	1.09	1.18	0.266	0.734	0.80	1.40	0.143	0.857
2.24	0.92	1.64	1.493	0.415	0.585	1.09	1.19	0.267	0.733	0.81	1.41	0.144	0.856
2.255	0.92	1.65	1.503	0.414	0.586	1.10	1.20	0.267	0.733	0.82	1.41	0.145	0.855
2.27	0.93	1.66	1.513	0.413	0.587	1.11	1.21	0.267	0.733	0.84	1.42	0.146	0.854
2.285	0.93	1.67	1.523	0.412	0.588	1.11	1.22	0.267	0.733	0.85	1.42	0.147	0.853
2.3	0.94	1.68	1.533	0.411	0.589	1.12	1.24	0.267	0.733	0.86	1.43	0.147	0.853
2.315	0.95	1.69	1.543	0.409	0.591	1.12	1.25	0.267	0.733	0.87	1.44	0.148	0.852
2.33	0.95	1.70	1.553	0.408	0.592	1.13	1.26	0.267	0.733	0.88	1.44	0.149	0.851
2.345	0.96	1.71	1.563	0.407	0.593	1.14	1.27	0.267	0.733	0.90	1.45	0.150	0.850
2.36	0.96	1.72	1.573	0.406	0.594	1.14	1.28	0.267	0.733	0.91	1.45	0.150	0.850
2.375	0.97	1.73	1.583	0.405	0.595	1.15	1.29	0.267	0.733	0.92	1.46	0.151	0.849
2.39	0.97	1.74	1.593	0.404	0.596	1.15	1.31	0.267	0.733	0.93	1.47	0.152	0.848
2.405	0.98	1.75	1.603	0.403	0.597	1.16	1.32	0.267	0.733	0.94	1.47	0.152	0.848
2.42	0.98	1.76	1.613	0.402	0.598	1.16	1.33	0.267	0.733	0.96	1.48	0.153	0.847
2.435	0.99	1.77	1.623	0.401	0.599	1.17	1.34	0.266	0.734	0.97	1.48	0.154	0.846

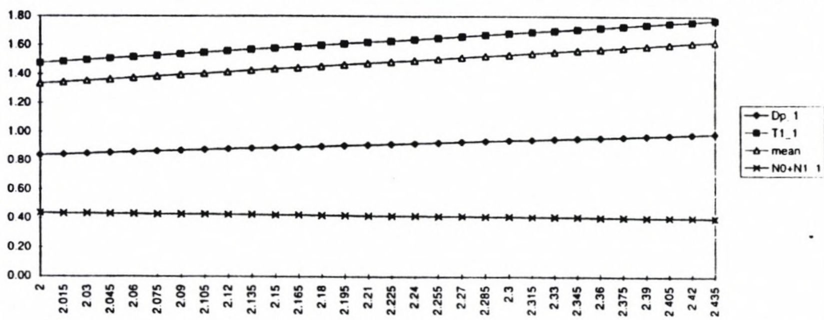
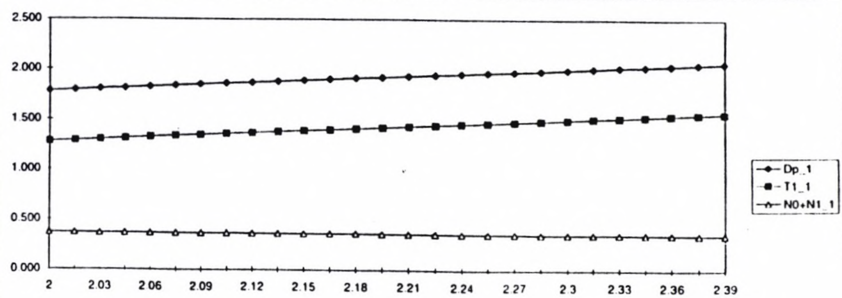


Table 17

$\Delta\alpha=0.5$	$\xi=1$											
	$\Delta\beta=1$				$\Delta\beta=1.5$				$\Delta\beta=2$			
τ	Δp_1	T1_1	N0+N1_1	NH_1	Δp_2	T1_2	N0+N1_2	NH_2	Δp_3	T1_3	N0+N1_3	NH_3
2	1.781	1.281	0.366	0.634	2.581	1.387	0.404	0.596	3.386	1.443	0.423	0.577
2.015	1.792	1.292	0.366	0.634	2.597	1.398	0.403	0.597	3.407	1.453	0.422	0.578
2.03	1.803	1.303	0.365	0.635	2.613	1.408	0.402	0.598	3.427	1.464	0.421	0.579
2.045	1.814	1.314	0.364	0.636	2.628	1.419	0.401	0.599	3.448	1.474	0.419	0.581
2.06	1.824	1.324	0.364	0.636	2.644	1.429	0.400	0.600	3.468	1.484	0.418	0.582
2.075	1.835	1.335	0.363	0.637	2.660	1.440	0.399	0.601	3.489	1.494	0.417	0.583
2.09	1.846	1.346	0.362	0.638	2.676	1.450	0.398	0.602	3.509	1.505	0.416	0.584
2.105	1.857	1.357	0.361	0.639	2.691	1.461	0.397	0.603	3.530	1.515	0.415	0.585
2.12	1.868	1.368	0.361	0.639	2.707	1.471	0.396	0.604	3.550	1.525	0.414	0.586
2.135	1.879	1.379	0.360	0.640	2.723	1.482	0.395	0.605	3.571	1.535	0.413	0.587
2.15	1.890	1.390	0.359	0.641	2.738	1.492	0.394	0.606	3.591	1.546	0.411	0.589
2.165	1.901	1.401	0.359	0.641	2.754	1.503	0.393	0.607	3.612	1.556	0.410	0.590
2.18	1.911	1.411	0.358	0.642	2.770	1.513	0.392	0.608	3.632	1.566	0.409	0.591
2.195	1.922	1.422	0.357	0.643	2.785	1.524	0.391	0.609	3.653	1.576	0.408	0.592
2.21	1.933	1.433	0.357	0.643	2.801	1.534	0.390	0.610	3.673	1.587	0.407	0.593
2.225	1.944	1.444	0.356	0.644	2.817	1.545	0.389	0.611	3.694	1.597	0.406	0.594
2.24	1.955	1.455	0.355	0.645	2.833	1.555	0.388	0.612	3.715	1.607	0.405	0.595
2.255	1.966	1.466	0.355	0.645	2.848	1.565	0.387	0.613	3.735	1.618	0.404	0.596
2.27	1.976	1.476	0.354	0.646	2.864	1.576	0.386	0.614	3.756	1.628	0.403	0.597
2.285	1.987	1.487	0.353	0.647	2.880	1.586	0.385	0.615	3.776	1.638	0.402	0.598
2.3	1.998	1.498	0.353	0.647	2.895	1.597	0.384	0.616	3.797	1.648	0.401	0.599
2.315	2.009	1.509	0.352	0.648	2.911	1.607	0.383	0.617	3.817	1.659	0.400	0.600
2.33	2.020	1.520	0.351	0.649	2.927	1.618	0.382	0.618	3.838	1.669	0.399	0.601
2.345	2.031	1.531	0.350	0.650	2.942	1.628	0.382	0.618	3.858	1.679	0.398	0.602
2.36	2.041	1.541	0.350	0.650	2.958	1.639	0.381	0.619	3.879	1.689	0.397	0.603
2.375	2.052	1.552	0.349	0.651	2.974	1.649	0.380	0.620	3.899	1.700	0.395	0.605
2.39	2.063	1.563	0.349	0.651	2.989	1.660	0.379	0.621	3.920	1.710	0.394	0.606



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